PHYSICAL REVIEW E VOLUME 62, NUMBER 4 OCTOBER 2000

Transmission regimes of periodic nonlinear optical structures

Dmitry E. Pelinovsky*
Department of Mathematics, University of Toronto, Toronto, Ontario, Canada M5S 3G3

Lukasz Brzozowski and Edward H. Sargent
Department of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario, Canada M5S 1A4

(Received 5 April 2000)

We investigate the input-output transmission regimes of optical structures with periodic nonlinear index. By deriving an analytical model from the Maxwell equations, we analyze the physical processes responsible for multistable and stable behavior. The threshold condition that separates multistable and stable transmission regimes is found exactly within the underlying model. We also derive analytical expressions for the limiting transmitted intensity in the stable regime and for the transmittance in the multistable regime in terms of optical wavelength and material parameters.

PACS number(s): 42.65.Pc, 42.79.Dj

Nonlinear periodic optical structures exhibit a response which is simultaneously wavelength- and intensity-dependent. The rapid transformation between states of low and high transmittance and the hysteresis in the input-output power relationship arise from the presence of optical bistability [1]. These physical processes make the nonlinear periodic structures be promising building blocks for functional, multiwavelength photonic systems. Application areas of bistable nonlinear gratings include optical signal processing [2–4], memory and logic [3], power limiting [4], bistable lasing [5], and beam reshaping [6].

Optical gratings are realized by modulating periodically the linear refractive index. A number of entirely new applications are enabled if not only the linear, but also the nonlinear, components of the refractive index can be managed [7]. These structures may exhibit stable limiting behavior in their input-output transmission characteristic: the transmitted intensity is clamped at the asymptotic limiting value and no switching to a state of higher transmittance takes place. This behavior, at once highly nonlinear yet stable, is conducive to all-optical limiting [7], sensor and personnel protection, logic, analog-to-digital conversion, and all-optical subtraction [8]. These concepts and applications are effective on both coherent and noncoherent optical signals.

Stable and bistable limiting regimes of nonlinear periodic structures are separated by a threshold that depends on material parameters and optical wavelength. This threshold has been widely studied for a nonlinear Fabry-Perot étalon [9], multilayer structures [10,11], and cascading materials based on backward second-harmonic generation [12]. In Fabry-Perot structures, the threshold for bistability was found in terms of the resonator finesse and nonlinear coefficient of the medium [13]. Li *et al.* gave the stability condition for nonlinear distributed feedback structures in terms of transmitted intensity [14]. He *et al.* suggested that the stability boundaries cannot in general be described by a simple relationship, but that low intensity states should yield stable solutions for any kind of structure [15].

In this Rapid Communication we describe a revolutionary way to achieve true all-optical limiting in the optical structure by periodic management of the Kerr nonlinearity. Moreover, we identify the exact analytical expression for the limiting intensity and for stability boundary separating the stable from the multistable transmission regime.

We investigate a particularly promising periodic optical structure that consists of alternating layers with matched linear refractive indices but different (positive versus negative) Kerr nonlinearities. The nonlinear periodic structure is shown in Fig. 1. The propagation of two noncoherent light waves of frequency ω is described by the Maxwell equations [16],

$$(U_+ + U_-)_{zz} + k^2[1+\Delta n(z) I(z)](U_+ + U_-) = 0. \quad (1)$$

Here $U_+(z)$ represents the forward (incident) wave, $U_-(z)$ the backward (reflected) wave, and the local intensity of light is $I(z) = |U_+(z)|^2 + |U_-(z)|^2$, found by averaging over the uncorrelated statistical ensemble [16]. The wave vector k and the optical wavelength λ (where $k = 2\pi n_0/\lambda$) are given within the linear theory as $k = \omega n_0/c$, where c is the speed of light in vacuum and n_0 is the linear index. The normalized nonlinear correction $\Delta n(z) = n_{nl}(z)/n_0$ depends on the Kerr coefficient $n_{nl}(z)$. The Kerr coefficient may be positive or negative depending on the medium and the wavelength [17].

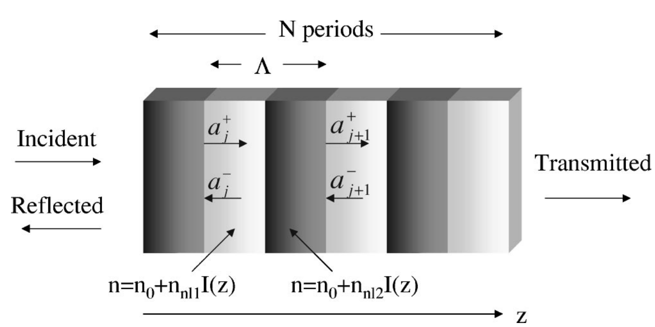

FIG. 1. Scheme of the periodic optical structure consisting of alternating layers with the same linear refractive index and different Kerr nonlinearities.

*Present address: Department of Mathematics, McMaster University, Hamilton, Ontario, Canada L8S 4K1.

We first use a scattering matrix approach [18]. We assume that the local intensity $I(z)$ is constant along each individual layer and absorption is negligible. The forward- and backward-propagating waves can then be represented explicitly at each layer (see Fig. 1) as

$$U_{\pm}(z)=\begin{cases} a_j^{\pm} e^{\pm ikz \pm ik\Delta n_{nl1} I_j (z-j\Lambda)}, & \text{layer I} \\ b_j^{\pm} e^{\pm ikz \pm ik\Delta n_{nl2} I_j (z-j\Lambda)}, & \text{layer II,} \end{cases} \quad (2)$$

where $I_j=|a_j^+|^2+|a_j^-|^2$ is the intensity, Λ is the period of the grating, N is the number of periods, and $0 \leqslant j \leqslant (N-1)$. The scattering matrix between $a_j^{\pm}$ and $a_{j+1}^{\pm}$ can be found from Eqs. (1) and (2) by matching the amplitudes and slopes of the electric field at the interface between two adjacent layers,

$$\begin{pmatrix} a_{j+1}^{+} e^{ik\Lambda} \\ a_{j+1}^{-} e^{-ik\Lambda} \end{pmatrix} = M_{21} M_{12} \begin{pmatrix} a_j^{+} \\ a_j^{-} \end{pmatrix}, \quad (3)$$

where the scattering matrix, e.g., M_{12}, is

$$M_{12}=\frac{1}{2}\begin{bmatrix} (1+k_1/k_2)e^{ik_1\Lambda/2} & (1-k_1/k_2)e^{-ik_1\Lambda/2} \\ (1-k_1/k_2)e^{ik_1\Lambda/2} & (1+k_1/k_2)e^{-ik_1\Lambda/2} \end{bmatrix},$$

and $k_{1,2}=k(1+\Delta n_{nl1,2} I_j)$. If the nonlinearity is small, i.e., $|\Delta n_{nl1,2}| I_j \ll 1$, the amplitudes $A^{\pm}(z_j)=a_j^{\pm}$ defined at $z=z_j=j\Lambda$ vary slowly across the adjacent layers. We can therefore assume

$$\lim_{\Lambda \to 0} \frac{A^{\pm}(z_{j+1})-A^{\pm}(z_j)}{\Lambda}=\frac{dA^{\pm}}{dz}$$

and derive coupled-mode equations in this slowly varying amplitude limit,

$$i\frac{dA^+}{dz}=k\overline{\Delta n_{nl}}(\kappa A^- e^{-3ik\Lambda/2}-A^+)(|A^+|^2+|A^-|^2), \quad (4)$$

$$i\frac{dA^-}{dz}=k\overline{\Delta n_{nl}}(-\kappa A^+ e^{3ik\Lambda/2}+A^-)(|A^+|^2+|A^-|^2), \quad (5)$$

where $\overline{\Delta n_{nl}}=(n_{nl1}+n_{nl2})/(2n_0)$ is the average normalized nonlinear index and κ is a product of variance of the nonlinear index and the resonance factor,

$$\kappa=\left|\frac{n_{nl1}-n_{nl2}}{n_{nl1}+n_{nl2}}\right| \frac{\sin(k\Lambda/2)}{k\Lambda/2}. \quad (6)$$

Exact resonance between the wave and the periodic grating occurs when $k\Lambda=\pi$, i.e., $\lambda=2\Lambda n_0$. The coupled mode model (4) and (5) for exact resonance and matched fluctuations of the nonlinear index (i.e., $n_{nl1}=-n_{nl2}$) was considered recently [7]. The model derived herein has wider applicability and describes wave propagation in the general nonlinear periodic structure with two alternating layers outside of the exact resonance case.

The nonlinear coupling between forward and backward waves is described by the κ terms in Eqs. (4) and (5). These terms provide stable, limiting behavior for $n_{nl1}=-n_{nl2}$, when $\kappa \to \infty$. The other right-hand-side terms in Eqs. (4) and (5) are associated with oscillatory behavior and, correspondingly, with multistability. Multistability finds its origins in the development of the cavity roundtrip phase between conditions of destructive and constructive interference as the average index evolves with intensity. The transition to multistability takes place when the self-coupling (destabilizing), oscillatory terms overwhelm the mutually coupling (limiting) terms. Here, we show that the threshold condition between these two regimes is given by $\kappa=1$, i.e., the stable limiting behavior occurs for

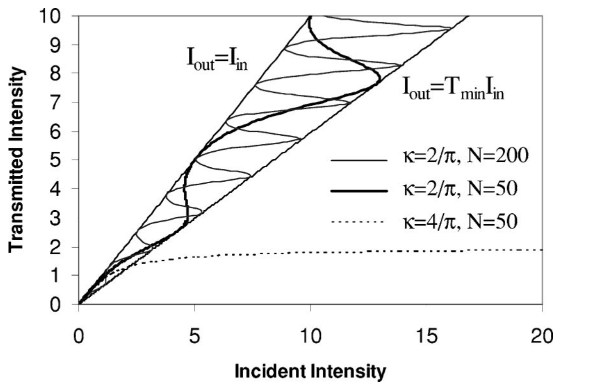

FIG. 2. Multistable and stable regimes of the nonlinear periodic structures at $k\Lambda=\pi$.

$$\left|\frac{n_{nl1}-n_{nl2}}{n_{nl1}+n_{nl2}}\right| \frac{\sin(k\Lambda/2)}{k\Lambda/2} \geqslant 1. \quad (7)$$

The coupled system (4) and (5) exhibits conservation of the energy flow through the optical structure,

$$|A^+(z)|^2-|A^-(z)|^2=I_{out}, \quad (8)$$

where $I_{out}=|A^+(l)|^2$ is the transmitted intensity at the right end of the structure, and $l=N\Lambda$ is the total length of the structure. There is no radiation incident on the structure from the right, which specifies the boundary condition: $A^-(l)=0$.

We show in Fig. 2 the transmitted ($I_{out}=|A^+(l)|^2$) versus incident ($I_{in}=|A^+(0)|^2$) intensity for two different structure lengths at exact resonance $k\Lambda=\pi$. The nonlinear indices are specified as $n_{nl1}=0.01$ and $n_{nl2}=0.00$ for two solid curves, where $\kappa=2/\pi$. This is the multistability regime when the transmitted intensity oscillates between the values determined by minimum and maximum transmittance,

$$T=1-\left|\frac{A^-(0)}{A^+(0)}\right|^2. \quad (9)$$

The maximum transmittance appears when $A^-(0)=0$, so that $T_{max}=1$. The minimum transmittance is defined by the condition $dA^-(0)/dz=0$, when $A^-(0)=\kappa A^+(0)e^{3ik\Lambda/2}$ so that $T_{min}=1-\kappa^2$. When $\kappa=0$, e.g., at $n_{nl1}=n_{nl2}$, the optical structure is homogeneous for all intensities and $I_{out}=I_{in}$. The greater is the parameter κ, the wider is the area between oscillations in the input-output transmission characteristics. We show in Fig. 2 that the period of the multistable oscillations (measured in terms of I_{in}) becomes smaller for

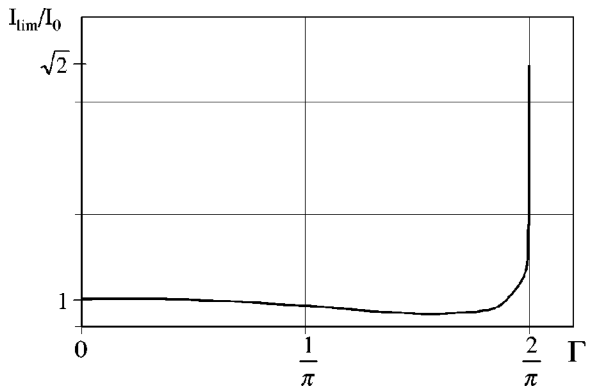

FIG. 3. Normalized limiting value of the output power I_{lim}/I_0 as a function of the inverse variance of the nonlinear index Γ at $k\Lambda=\pi$.

longer structures (when N grows). As a result, more possible transmission levels are present within a given range of the incident intensity.

When κ reaches 1, T_{min} vanishes. This marks the onset of true, stable optical limiting. In the region $\kappa \geqslant 1$, the cross-coupling of two waves dominates over the phase-related oscillations and the multistability regime is replaced by the stable limiting transmission regime. We show the stable limiting behavior by a dashed curve in Fig. 2 for the parameter: $n_{nl1}=0.015$, $n_{nl2}=-0.005$, when $\kappa=4/\pi$.

In order to find the limiting value for transmitted intensity and to characterize the features of the multistability regime, we construct exact solutions to Eqs. (4) and (5). First, we rescale the distance z by $Z=k\Delta\overline{n}_{nl}z$ and substitute the amplitudes $A^{\pm}(z)$ in the polar form,

$$A^{+}(z)=\sqrt{I_{out}+Q}e^{i(\Phi+\Psi)}, \tag{10}$$

$$A^{-}(z)=\sqrt{Q}e^{i(\Phi+3k\Lambda/2)}. \tag{11}$$

Here $Q(Z)$ and $\Phi(Z)$ are the intensity and the complex phase of the reflected wave, respectively, and $\Psi(Z)$ is the phase mismatch between the incident and reflected waves. The coupled system (4) and (5) reduces to the following form:

$$\frac{dQ}{dZ}=-2(I_{out}+2Q)\sqrt{Q(I_{out}+Q)}\kappa\sin\Psi, \tag{12}$$

$$\frac{d\Psi}{dZ}=(I_{out}+2Q)\left[2-\frac{I_{out}+2Q}{\sqrt{Q(I_{out}+Q)}}\kappa\cos\Psi\right]. \tag{13}$$

The boundary conditions are $Q(L)=0$ and $\Psi(L)=\pi/2$, where $L=k\Delta\overline{n}_{nl}l$. The latter condition follows from Eq. (13) as $Q(L)$ vanishes and from Eq. (12) as $Q(Z)$ has a negative slope near $Z=L$. Subject to this boundary condition, we find the integral of Eqs. (12) and (13) in the form,

$$\kappa\cos\Psi=\sqrt{\frac{Q}{I_{out}+Q}}\geqslant 0. \tag{14}$$

Using this relation the system (12) and (13) can be reduced to the single equation,

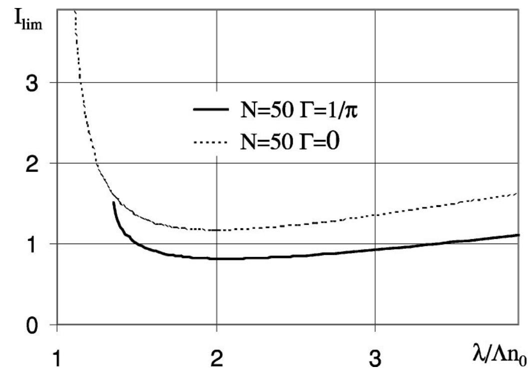

FIG. 4. Limit transmitted power I_{lim} as a function of the wavelength ratio $\lambda/(\Lambda n_0)$.

$$\frac{d\Psi}{dZ}=I_{out}[1+\kappa^2\cos^2\Psi], \tag{15}$$

which can be further integrated. It is obvious from Eqs. (14) and (15) that $\Psi(Z)$ always increases from $\Psi(0)$ to $\Psi(L)=\pi/2$. However, since $\cos\Psi$ is non-negative, the phase $\Psi(Z)$ may have jumps from $\Psi=\pi/2$ to $\Psi=-\pi/2$ at the points inside the interval $0<Z<L$, where $Q(Z)$ vanishes. Only the fundamental branch of solutions has no jumps and this branch is unique in the limiting transmission regime.

The exact solution for $Q(Z)$ follows from Eqs. (14) and (15) in the form

$$Q(Z)=\frac{\kappa^2 I_{out}\sin^2[\sqrt{1+\kappa^2}I_{out}(L-Z)]}{1+\kappa^2\cos[2\sqrt{1+\kappa^2}I_{out}(L-Z)]}. \tag{16}$$

We show from Eq. (16) that the two transmission regimes are separated by the condition $\kappa=1$.

In the multistable regime, $\kappa<1$, the solution (16) is nonsingular for any value of I_{out}. The transmittance T can be found from Eqs. (9) and (16) in the form

$$T=\frac{1+\kappa^2\cos[2\sqrt{1+\kappa^2}I_{out}L]}{1+\kappa^2\cos^2[\sqrt{1+\kappa^2}I_{out}L]}. \tag{17}$$

The points of maximum transmittance $[T_{max}=1,\ Q(0)=0]$ are given by the roots

$$I_{out}=I_n=\frac{\pi n}{\sqrt{1+\kappa^2}L},\quad n=0,1,2,\ldots\ . \tag{18}$$

The distribution for the reflected wave $Q(Z)$ has exactly n nodes across the optical structure within the parameter range $I_n\leqslant I_{out}<I_{n+1}$. For each node, the phase $\Psi(Z)$ jumps from $+\pi/2$ to the left of the node to $-\pi/2$ to the right. The points of minimum transmittance $[T_{min}=1-\kappa^2,\ dQ(0)/dZ=0]$ are located exactly in the middle of each interval (I_n,I_{n+1}).

In the stable regime, $\kappa\geqslant 1$, the distribution of the reflected wave $Q(Z)$ becomes singular at $I_{out}\geqslant I_{lim}$, where

$$I_{lim}=\frac{\pi}{4\sqrt{1+\kappa^2}L}\left[1+\frac{2}{\pi}\arcsin\left(\frac{1}{\kappa^2}\right)\right]. \tag{19}$$

At the limiting value, $I_{out}=I_{lim}$, the distribution $Q(Z)$ diverges as $Z\to 0$ so that $I_{in}\to\infty$. True optical limiting is therefore achieved: the transmitted intensity is bounded by its limiting value irrespective of the incident power (see the dashed curve in Fig. 2).

When $\overline{n_{nl}}\to 0$, the limiting intensity approaches the asymptotic value (see also [7]),

$$\lim_{\overline{n_{nl}}\to 0} I_{\rm lim}=I_0=\frac{\pi n_0}{4N|n_{nl1}-n_{nl2}|\sin(k\Lambda/2)}. \tag{20}$$

We plot in Fig. 3 the normalized limiting intensity (I_{lim}/I_0) as a function of Γ at the exact resonance $k\Lambda=\pi$, where Γ is the inverse variance of the nonlinear index given by

$$\Gamma=\left|\frac{n_{nl1}+n_{nl2}}{n_{nl1}-n_{nl2}}\right|. \tag{21}$$

When the inverse variance Γ is small, the normalized limiting intensity is smaller than 1. When Γ approaches the threshold boundary (7), i.e., $\Gamma=2/\pi$ for the exact resonance, the normalized intensity approaches $\sqrt{2}$. Thus, the limiting intensity remains within 40% of its asymptotic value I_0 for any value of the material parameters.

The stable limiting regime of the periodic optical structure is supported by a low average Kerr coefficient throughout the structure accompanied by a high layer-to-layer variance. It is facilitated by close proximity to the Bragg resonance. When the light wavelength λ deviates from the exact resonance $\lambda=2\Lambda n_0$, the stable regime breaks down. We illustrate this feature in Fig. 4 by plotting the limiting transmitted intensity I_{lim} (19) versus the wavelength ratio $\lambda/(\Lambda n_0)$ for two values of Γ: $\Gamma=0$ (dashed curve) and $\Gamma=1/\pi$ (solid curve). The stable behavior of the nonlinear periodic structure is affected weakly by deviation of the light wavelength to longer-than-resonance region, while shorter-than-resonance wavelengths quickly undergo transitions to the multistable regime (see Fig. 4).

In conclusion, we have elaborated and explained the conditions for stability and true asymptotic limiting in nonlinear periodic structures. Stable all-optical limiting is a highly promising avenue towards optical signal processing. We have derived a threshold condition that predicts, in terms of the material parameters and optical wavelength, whether a given structure is stable or multistable.

[1] H. M. Gibbs, *Optical Bistability: Controlling Light with Light* (Academic Press, Orlando, 1985).
[2] P. Tran, J. Opt. Soc. Am. B **16**, 70 (1999).
[3] P. W. Smith, IEEE Spectr. **18**, 26 (1981).
[4] C. J. Herbert, W. S. Capinski, and M. S. Malcuit, Opt. Lett. **17**, 1037 (1992); M. Scalora, J. P. Dowling, C. M. Bowden, and M. J. Bloemer, Phys. Rev. Lett. **73**, 1368 (1994).
[5] J-Y. Wang, M. Cada, R. Van Dommelen, and T. Makino, IEEE J. Quantum Electron. **3**, 1271 (1997); J. Zhou, M. Cada, and T. Makino, J. Lightwave Technol. **15**, 342 (1997); D. N. Maywar and G. P. Agrawal, IEEE J. Quantum Electron. **33**, 2029 (1997).
[6] X. Lu, Y. Bai, S. Li, and T. Chen, Opt. Commun. **156**, 219 (1998).
[7] L. Brzozowski and E. H. Sargent, J. Opt. Soc. Am. B **17**, 1360 (2000).
[8] L. Brzozowski and E. H. Sargent, IEEE J. Quantum Electron. **36**, 550 (2000).
[9] R. S. Jameson and W-T. Lee, IEEE J. Quantum Electron. **25**, 139 (1989); J. M. Halley and J. E. Midwinter, *ibid.* **26**, 348 (1990).
[10] S. D. Gupta, Phys. Rev. B **38**, 3628 (1988); J. He and M. Cada, IEEE J. Quantum Electron. **27**, 1182 (1991); J. Danckaert, K. Fobelets, and I. Veretennicoff, Phys. Rev. B **44**, 8214 (1991).
[11] H. G. Winful, J. H. Marburger, and E. Garmire, Appl. Phys. Lett. **35**, 379 (1979); A. Mecozzi, S. Trillo, and S. Wabnitz, Opt. Lett. **12**, 1008 (1987).
[12] M. Matsumoto and K. Tanaka, IEEE J. Quantum Electron. **31**, 700 (1995); G. D'Alessandro, P. S. J. Russel, and A. A. Wheeler, Phys. Rev. A **55**, 3211 (1997); C. Conti, F. Assanto, and S. Trillo, Opt. Lett. **24**, 1139 (1999).
[13] C-F. Li and A-Q. Ma, *Optical Bistability 2*, edited by C. M. Bowden, H. M. Gibbs, and S. McCall (Plenum Press, New York, 1984), Vol. 167.
[14] Q. Li, C. T. Chan, K. M. Ho, and C. M. Soukoulis, Phys. Rev. B **53**, 1557 (1996).
[15] H. He, A. Arraf, C. M. de Sterke, P. D. Drummond, and B. A. Malomed, Phys. Rev. E **59**, 6064 (1999).
[16] B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics* (John Wiley and Sons, New York, 1991).
[17] R. Rangel-Rojo, S. Yamada, S. Matsuda, and H. D. Yankelevich, Appl. Phys. Lett. **72**, 1021 (1998); H. S. Nalwa, and S. Miyata, *Nonlinear Optics of Organic Molecules and Polymers* (CRC Press, Boca Raton, FL, 1999).
[18] P. Yeh, *Optical Waves in Crystals* (John Wiley and Sons, New York, 1984).