Optimizing Spectral Efficiency in Multiwavelength Optical CDMA System

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***Abstract*—Prime codes are excellent candidates for use in multiwavelength optical code-division multiple-access (O-CDMA) systems. We show that in an O-CDMA system using two-dimensional single-pulse-per-row codes, a single choice of the number of wavelength channels can accommodate different numbers of users with maximal spectral efficiency. The optimum single-user-detection spectral efficiency of the system can be reached using AND detection. A fixed-hardware network can readily be adapted in response to changes in the number of users and traffic load.**

***Index Terms*—Adaptive networks, optical code-division multiple access (O-CDMA), spectral efficiency, two-dimensional (2-D) codes.**

I. Introduction

OPTICAL code-division multiple access (O-CDMA) has attracted attention for over a decade [1]–[3]. While the vast bandwidth of the optical fiber medium provides high-speed point-to-point data transmission, the CDMA scheme facilitates random access to the channel in a bursty traffic environment.

In view of the nonnegativity constraint of the incoherent optical channel, conventional electronic codes cannot be directly applied. While optical codes with good correlation properties have been developed using exclusively the time axis [1], [2], [4], the spectral (wavelength channel) domain has emerged as an additional readily exploitable degree of freedom in the development of pseudoorthogonal codes. Two-dimensional (2-D) codes, spread along both the temporal and either the wavelength-channel or spatial axes, have received particular attention [5]–[8]. In particular, prime codes have been the focus of research in new codes for asynchronous sharing of a fiber-optic medium [9], [10]. The prime codes have been shown to support all network users simultaneously with a bit-error rate (BER) of less than 10^{-9} while employing significantly shorter codes than optical orthogonal codes [9]. Prime codes' low-shifted autocorrelation $(\lambda_a = 0)$ ensures synchronizability. A low crosscorrelation (e.g., $\lambda_c = 1$) suppresses multiple-access interference (MAI) and thereby permits simultaneous access to the channel by many users.

While the prime codes have been shown to have tremendous potential utility, the design of multiuser networks for optimal spectral efficiency using prime codes remains to be explored. Although CDMA systems generally have very low spectral efficiency, it is of engineering importance, as well as fundamental interest, to identify the maximum spectral efficiency achievable using CDMA on nonnegative optical channel.

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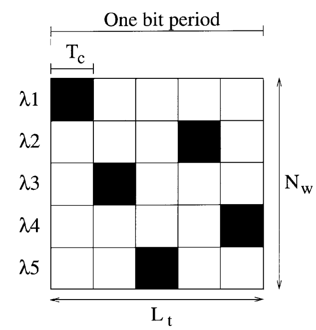

Fig. 1. An example of SPR prime code from [9]. In this case, $N_W = 5, L_t = 5$, and $S = 25$. T_c is a time duration of a chip.

In this letter, we explore the effect of distribution of bandwidth between the temporal and wavelength/spatial domains on the spectral efficiency of the system. Next, we investigate the effect of detection method on the achieved spectral efficiency of the system. Finally, we summarize the importance and applications of the results.

II. Bandwidth Allocation

While the main motivation for using O-CDMA is not its efficiency of bandwidth utilization—the focus is rather on creating a low-cost, decentralized and simple high-speed multiple-access environment—spectral efficiency can nevertheless be one of a number of metrics considered in system design.

In the case of the single-pulse-per-row (SPR) prime code family of [9], a code set is constructed using N_W wavelengths and L_t time chips in one bit period per wavelength. An example of this is shown in Fig. 1. The darkened squares indicate the locations of optical pulses in a code. They are called the "1" chips of the code.

We estimate the upper bound on the BER of a system in which MAI is the dominant BER-limiting mechanism. From [9], for O-CDMA systems using 2-D SPR codes, the BER due to MAI is given by

$$\mathrm{BER} = \frac{1}{2}\sum_{i=N_W}^{N_{\mathrm{su}}-1}\binom{N_{\mathrm{su}}-1}{i}\left(\frac{N_W}{2L_t}\right)^i\left(1-\frac{N_W}{2L_t}\right)^{N_{\mathrm{su}}-1-i} \tag{1}$$

where N_{su} is the number of simultaneous users in the system, L_t is the temporal length employed, and N_W is the number of wavelength channels used. We define the spectral efficiency η of an O-CDMA system as

$$\eta = \frac{\text{Aggregate Information Rate}}{\text{Total Spectral Bandwidth}} = \frac{N_{\mathrm{su}}R_B}{N_W\Delta f_{\mathrm{ch}}} \tag{2}$$

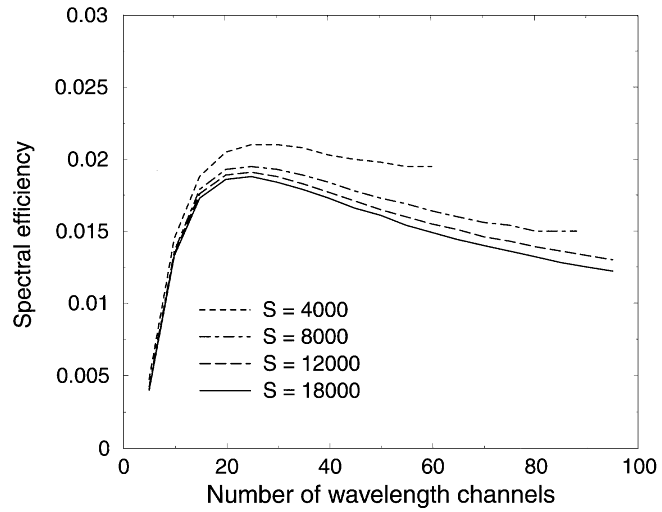

Fig. 2. Spectral efficiency versus number of wavelength channels for different choices of spreading factor S with a BER ceiling of 10^{-9}.

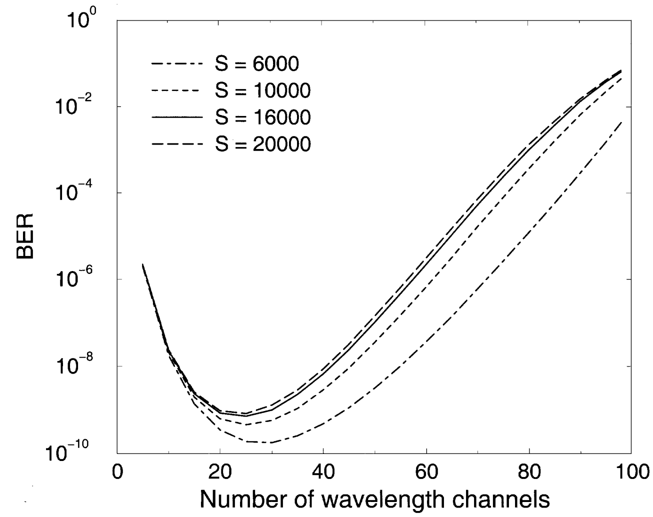

Fig. 3. BER for a spectral efficiency of 0.0185 versus the number of wavelength channels for different choices of spreading factor S.

where R_B is the bit rate per user and $\Delta f_{\rm ch}$ is the bandwidth of each wavelength channel. The total bandwidth expansion of the 2-D CDMA code is the combined effect of expansion along the temporal and wavelength-channel axes. To quantify this total bandwidth expansion, we define the term spreading factor S as $S = N_W L_t$.

Using (1), we plot in Fig. 2 the spectral efficiency against the number of wavelength channels used in the system for a required BER of 10^{-9} or better.

Within each curve, we fix the total bandwidth expansion (S) and vary the relative allocation of bandwidth in time (L_t) and wavelength (N_W).

As the spreading factor is increased, Fig. 2 shows that the maximum achievable spectral efficiency converges to a single level. The number of wavelength channels which can achieve this maximum spectral efficiency also converges to a single value. Specifically, for BER $= 10^{-9}$, the choice of the number of wavelength channels which gives the highest efficiency converges to 24. This result contradicts one possible intuition—the equal allocation of bandwidth among the temporal and wavelength/spatial domains in a 2-D code can always achieve the best efficiency.

In order to gain more insight into this phenomenon, we use (1) again to plot in Fig. 3 the BER achieved against the number of wavelength channels employed in the system for a fixed spectral efficiency.

In Fig. 3, this fixed spectral efficiency used is the converged value of peak efficiency, the highest efficiency achieved in the limit of large spreading factor, obtained from Fig. 2.

In each curve, we once again fix the total bandwidth expansion (the spreading factor S) and change the relative allocation between wavelength (N_W) and time (L_t). As the spreading factor is increased, the minimum BER each curve achieves converges to a single value. In this case, the minimum BER reaches 10^{-9}, which is the upper-bound BER we set in Fig. 2. The number of wavelengths at which this minimum BER occurs also converges to 24. We also note from Fig. 3 that the BER curves for spreading factors of 16 000 and 20 000 almost overlap with each other. This confirms the convergence behavior of the system performance for large spreading factor.

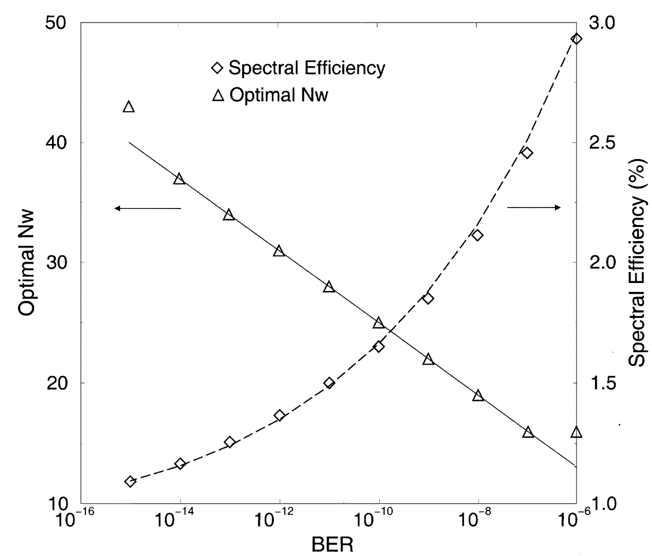

Fig. 4. Converged value of spectral efficiency η and the corresponding optimal number of wavelengths N_W for different BERs when the spreading factor is large. Values obtained from observing the convergence are shown as symbols only. The approximation curves from (3) and (4) are shown as lines.

By deploying the above techniques of analysis to a range of BER, we explore the evolution of the converged value of spectral efficiency η and the corresponding optimal N_W for large spreading factor. The values obtained from observing the convergence of the optimal N_W and spectral efficiency η are summarized in Fig. 4.

We can approximate the trend of the observed optimal N_W points (Δ) on Fig. 4 by

$$N_W = -3\log(\text{BER}) - 5. \tag{3}$$

Likewise, the trend of the observed spectral efficiency η points $(\diamond)$ on Fig. 4 can also be approximated by

$$\sqrt{\log(\eta)} = 0.0541\log(\text{BER}) + 1.0108 \tag{4}$$

where η is in percentage. In Fig. 4, we show the close agreement between the above approximations and the observed data points.

From (3) and (4), it can be seen that once the BER is chosen, the optimal N_W and the corresponding η are uniquely determined. We confirm this observation by an independent analysis in the Appendix. We show that for large spreading factors, the converged value of N_W at which the minimum BER occurs is given by the solution of

$$\left(\frac{-1}{1250}\right) N_W{}^3 - \left(\eta - \frac{2}{25}\right) N_W{}^2 + \left(\ln\left(\frac{\eta}{2}\right) + \ln(25) + 0.5\right) N_W - \frac{1}{2} = 0. \quad (5)$$

We see from (5) that the solutions to the optimal N_W value depend solely on η, which, in turn, depends exclusively on the BER chosen. These analytically derived results agree with our results of (3) and (4).

The bit rate per user R_B in (2) can be approximated by $(\Delta f_{\text{ch}}/L_t)$ when a practical optical pulse is considered. Then, (2) can be written as

$$\eta = \frac{N_{\text{su}}}{N_W L_t}. \quad (6)$$

Given a fixed BER, the converged values of η and the corresponding optimal N_W are uniquely determined. Then, the value of L_t depends exclusively on N_{su}. This direct proportionality relationship between L_t and N_{su} for a fixed BER allows the network controller to regulate the network error performance conveniently. As the number of active users N_{su} in the network changes, the BER of the network can be maintained simply by adjusting the temporal length L_t of the system.

III. Detection Method

In the analysis of Section II, it is assumed that a bit "1" is detected at the receiver of each node when the sum of the light intensities detected at the N_W "1" chip positions is at least N_W. This detection method is denoted SUM detection. It is shown that the optimum single-user detection method for systems using 2-D optical CDMA codes is AND detection [11]. For 2-D SPR codes, AND detection means that a bit "1" is detected at the receiver of each node only if the light intensity detected at each of the N_W "1" chip positions is at least N_W. It is further shown that the spectral efficiency of a system using 2-D O-CDMA code can be at least doubled by replacing the conventional SUM detection method with the AND detection one. To obtain the spectral efficiency of 2-D SPR codes with AND detection, we multiply the η values on Fig. 4 by the spectral efficiency improvement factors for different BER shown in [11]. The spectral efficiencies of the 2-D SPR codes with AND detection and with SUM detection are compared in Fig. 5.

Also in Fig. 5, we show the spectral efficiency of baseband detection method of bipolar O-CDMA code (adopted from [12]). The spectral efficiency of AND detection of 2-D SPR code is very close to that of the baseband bipolar code. This shows that AND detection of deterministic 2-D codes can perform as well as baseband bipolar CDMA codes.

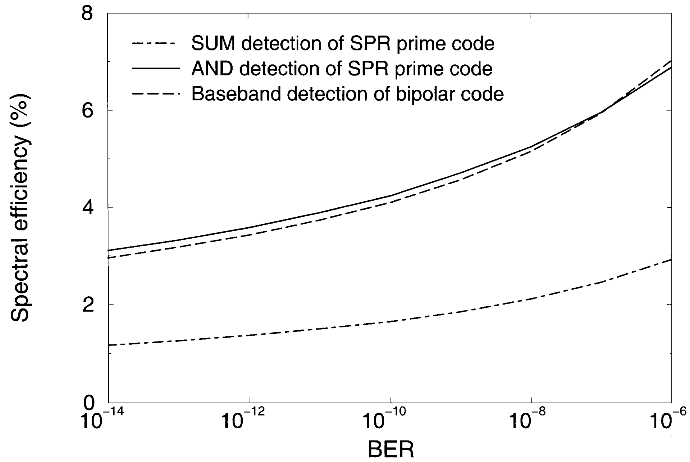

Fig. 5. Spectral efficiency achieved using SUM detection and AND detection on 2-D SPR codes for different BERs. For comparison, the spectral efficiency of baseband bipolar CDMA codes is also shown.

In sum, given a fixed BER, the optimum single-user-detection spectral efficiency of a 2-D SPR code system can be achieved using the AND detection method with a single choice of the number of wavelengths N_W.

IV. Conclusions

In a multiwavelength O-CDMA system using SPR codes, the maximum spectral efficiency of the code converges to a fixed level as the total bandwidth expansion of the code set increases. Using the AND detection technique, rather than the SUM, the spectral efficiency of a 2-D SPR code system is as high as that of baseband bipolar CDMA systems for a broad range of BERs.

As the maximum spectral efficiency converges to a fixed value, the number of wavelength channels at which this maximum occurs also converges to a value which is independent of the time spreading. These converged values depend exclusively on the choice of BER. Our results imply that, once the acceptable BER has been determined (based, for example, on user requirements as well as the error detection/correction codes and protocols available), a single choice of the number of wavelength channels suffices in accommodating different numbers of users with maximum spectral efficiency.

The network can thus be made adaptive in two ways.

- A readily scalable network. The SPR codes considered permit the introduction of additional users through increased time spreading for a given number of wavelength channels.
- A time-dependent network. The code set can be dynamically readjusted (again, with fixed hardware, and therefore, fixed number of wavelengths) in order to accommodate more or less simultaneous users, depending on the traffic load over any given interval.

Appendix

To fix the spreading factor S, we employ the substitution

$$L_t = \frac{S}{N_W}. \quad \text{(A.1)}$$

Then, (1) becomes a function of $N_{\rm su}$, N_W, and S only

$$\mathrm{BER} = \frac{1}{2}\sum_{i=N_W}^{N_{\rm su}-1}\binom{N_{\rm su}-1}{i}\left(\frac{{N_W}^2}{2S}\right)^i\left(1-\frac{{N_W}^2}{2S}\right)^{N_{\rm su}-1-i}. \quad (A.2)$$

Close examination suggests that (A.2) is dominated by the first term of the summation for N_W less than the ratio $S/N_{\rm su}$. (A.2) can be simplified to

$$\mathrm{BER} = \frac{1}{2}\binom{N_{\rm su}-1}{N_W}\left(\frac{{N_W}^2}{2S}\right)^{N_W}\left(1-\frac{{N_W}^2}{2S}\right)^{N_{\rm su}-1-N_W}. \quad (A.3)$$

After taking natural logarithm on both sides of (A.3), we apply Stirling's formula $(\ln(n!) \approx n\ln(n) - n)$ to the factorial terms. Further simplifications using the Taylor series expansion lead to

$$\begin{aligned}\ln(\mathrm{BER}) = &\frac{{N_W}^3}{2S} + \left(\frac{1-N_{\rm su}}{2S}\right){N_W}^2 + \ln\left(\frac{{N_W}^2}{2S}\right)N_W \\ &+ (N_W - N_{\rm su} + 0.5)\ln(N_{\rm su} - N_W - 1) \\ &- (N_W + 0.5)\ln(N_W). \quad (A.4)\end{aligned}$$

Taking the first derivative of (A.4) with respect to N_W and setting it to zero, we obtain

$$\begin{aligned}&\left(\frac{3}{2S} - \frac{1}{2}\left(\frac{1}{N_{\rm su}-1}\right)^2\right){N_W}^3 - \\ &\left(\frac{N_{\rm su}-1}{S} + \frac{1}{N_{\rm su}-1}\right){N_W}^2 + \\ &\left(\ln\frac{N_{\rm su}-1}{2S} + ln(N_W) + 2\right)N_W - \\ &\frac{N_W}{2(N_W - N_{\rm su} + 1)} - 0.5 = 0. \quad (A.5)\end{aligned}$$

The first and the second-to-last terms of (A.5) are negligible relative to others, so that (A.5) may be simplified to yield

$$\begin{aligned}-&\left(\frac{N_{\rm su}-1}{S} + \frac{1}{N_{\rm su}-1}\right){N_W}^2 + N_W\ln(N_W) \\ &+ \left(\ln\left(\frac{N_{\rm su}-1}{2S}\right) + 2\right)N_W - 0.5 = 0. \quad (A.6)\end{aligned}$$

For a BER range of 10^{-6} to 10^{-12}, Fig. 4 indicates that the value of the optimal N_W ranges from 20 to 30. This means that the mean m of N_W in this range is 25. When we apply this concept of mean into the second term of (A.6), we get with Taylor series expansion

$$N_W\ln(N_W) = N_W(\ln(m) - 1.5) + \frac{2}{m}{N_W}^2 - \frac{1}{2m^2}{N_W}^3. \quad (A.7)$$

Using (A.7), (A.6) becomes

$$\begin{aligned}&\left(\frac{-1}{2m^2}\right){N_W}^3 - \left(\frac{N_{\rm su}-1}{S} + \frac{1}{N_{\rm su}-1} - \frac{2}{m}\right){N_W}^2 \\ &+ \left(\ln\left(\frac{N_{\rm su}-1}{2s}\right) + \ln(m) + \frac{1}{2}\right)N_W - \frac{1}{2} = 0. \quad (A.8)\end{aligned}$$

We fix the ratio of $N_{\rm su}/S$ and replace it by η. If we let S, and hence, $N_{\rm su}$ tend to infinity, (A.8) becomes

$$\begin{aligned}&\left(\frac{-1}{1250}\right){N_W}^3 - \left(\eta - \frac{2}{25}\right){N_W}^2 \\ &+ \left(\ln\left(\frac{\eta}{2}\right) + \ln(25) + \frac{1}{2}\right)N_W - \frac{1}{2} = 0. \quad (A.9)\end{aligned}$$

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