Above-Threshold Leakage in Semiconductor Lasers: An Analytical Physical Model

Igor M. P. Aarts and Edward H. Sargent

Abstract—We present an analytical physical model for above-threshold leakage in semiconductor lasers. The model can be applied to estimate whether heterobarrier lowering and accompanying overbarrier leakage are within reach of having serious deleterious effects on laser performance. The model uses two-dimensional fully self-consistent numerical equations that arise from comprehensive systems of partial coupled differential equations. The effect of temperature and doping on laser efficiency is analyzed for two lasers, one designed for operation at 1.3 μm and the other at 1.55 μm. Both devices are assumed to be built in the InGaAsP-InP material system. We show that, even in a 1.55-μm laser, overbarrier leakage can cause severe performance degradation at typical operating temperatures and doping levels, and we argue that overbarrier leakage deserves to be treated as a potential threat to laser performance at telecommunication wavelengths.

Index Terms—Analytical model, doping, internal efficiency, overbarrier leakage, semiconductor laser, telecommunication, temperature.

I. INTRODUCTION

The above-threshold efficiency of semiconductor lasers emitting light at 1.3 μm and 1.55 μm has attracted significant attention over the past twenty years. The use of lasers in CATV distribution systems and in emerging metropolitan-area networks further exacerbates the need for efficient operation of uncooled lasers. By eliminating the need for active cooling, the development of uncooled lasers achieves cost reduction, low power consumption, and compactness [1].

Westbrook and Nelson [2] proposed an analytical model for electron leakage in 1.5-μm separate-confinement heterostructure lasers but neglect the role of leakage of active region carriers over the heterobarrier into the minority contact (overbarrier leakage). Kazarinov and Pinto [3] quantified explicitly the role of overbarrier leakage in double-heterostructure semiconductor lasers. They explored the role of injected forward current in lowering the confinement barrier for the electrons. In their model, the authors considered simultaneously Poisson's equation, current continuity for electrons and holes, drift and diffusion, Auger recombination, Shockley–Read–Hall recombination, and spontaneous and stimulated emission terms. They explored the role of forward current and temperature on thermionic emission of electrons from the active region and, consequently, on internal quantum efficiency.

If the mechanism of heterobarrier lowering using fully self-consistent numerical simulation [3], [4] and reported experimentally in [5] can, indeed, represent a significant cause of efficiency degradation, then the laser designer will benefit greatly by having a simplified, direct, analytical model, and correspondingly straightforward physical understanding of this effect. Before proceeding to obtain two-dimensional (2-D) fully self-consistent numerical solutions to the comprehensive system of coupled partial differential equations, a simplified model could be applied to estimate whether heterobarrier lowering and accompanying overbarrier leakage are indeed within reach of having serious deleterious effects on laser performance.

II. MODEL

We seek in this work to consider the role of overbarrier leakage in isolation from all other mechanisms potentially responsible for above-threshold efficiency degradation. Such other effects include parallel leakage, injection-level-dependent free carrier absorption, and Auger recombination and associated carrier heating. We take each of these effects to be a parameter in our model. Thus, full consideration of all such mechanisms combined is readily achieved by parameterizing each according to its dependencies on structure and bias. In this work, we consider instead the direct impact of overbarrier leakage on efficiency degradation assuming that all other efficiency-degrading mechanisms remain fixed above the lasing threshold.

We begin with [3, eq. (4)]

\[ q\Delta V = kT \ln \left( 1 + \frac{J_{\text{act}} \cdot L_D \exp\left( \frac{\Delta E_V - \Delta E_{Fp} - \Delta E_{Fn}}{kT} \right)}{\sqrt{2\mu_p N_A \sqrt{V_1 kT}}} \right) \]

(1)

The meaning of each symbol is explained in Table I and illustrated in Fig. 1. In this work, except when stated otherwise, we use the material parameters given in Table I, obtained from [6], for bulk lasers operating at 1.3 μm.

The effects explored in this work are important in both bulk and quantum-well (QW) lasers. In the quantitative portions of this work, we consider for illustrative purposes the case of bulk lasers. This approach applies to the case of overbarrier leakage from the (bulk) separate-confinement heterostructure (SCH) region of a QW laser as well. To consider leakage directly from
TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$      electron charge</td>
<td>1.6x10^19 C</td>
</tr>
<tr>
<td>$k$      Boltzmann’s constant</td>
<td>1.38x10^23 J/K</td>
</tr>
<tr>
<td>$T$      temperature</td>
<td>300 K</td>
</tr>
<tr>
<td>$\mu_p$  hole mobility (at 300 K)</td>
<td>200 cm^2/Vs</td>
</tr>
<tr>
<td>$\mu_n$  electron mobility (at 300 K)</td>
<td>3000 cm^2/Vs</td>
</tr>
<tr>
<td>$n_{act}$ electron density in active region</td>
<td>2x10^5 m^-3</td>
</tr>
<tr>
<td>$\tau_e$ electron lifetime in p-contact</td>
<td>10^-9 s</td>
</tr>
<tr>
<td>$N_A$    acceptor concentration in depletion layer</td>
<td>4x10^17 cm^-3</td>
</tr>
<tr>
<td>$\sigma_p$ electrical conductivity of the p-cladding layer</td>
<td>$q\mu_p$</td>
</tr>
<tr>
<td>$L_D$    extrinsic Debye length</td>
<td>$\frac{kT \epsilon}{4\pi N_A q^2}$</td>
</tr>
<tr>
<td>$D_n$    diffusion coefficient</td>
<td>$\frac{\mu_k T}{q}$</td>
</tr>
<tr>
<td>$L_n$    electron diffusion length</td>
<td>$\sqrt{D_n \tau_e}$</td>
</tr>
<tr>
<td>$L_m$    electron drift length</td>
<td>$\frac{kT \sigma_p}{q J_{act}}$</td>
</tr>
<tr>
<td>$\Delta E_V$ discontinuity of the valence band</td>
<td>0.24 eV</td>
</tr>
<tr>
<td>$\Delta E_{FR}$ difference between conduction band edge and electron Fermi level energy</td>
<td>$\approx kT$</td>
</tr>
<tr>
<td>$\Delta E_{HF}$ difference between valence band edge and hole Fermi level energy</td>
<td>$\approx kT$</td>
</tr>
<tr>
<td>$q\Delta V$ change in hole quasi-Fermi level at heterojunction</td>
<td>$0.317$ eV</td>
</tr>
<tr>
<td>$V_1$    potential drop in depletion layer</td>
<td>$V_{bd} - V_{act} = V_{bf} + \Delta V$</td>
</tr>
</tbody>
</table>

QW states to a bulk material, it is necessary to modify the effective density of states used in the relation between carrier density and quasi-Fermi energy.

As explained in [3], the quantity $V_1$ represents the potential drop in the depletion layer. This quantity is in turn related to the properties of the heterostructure at equilibrium and to the degree of forward injections:

$$V_1 = V_{bd} - V_{act} = V_{bf} + \Delta V$$  (2)

where $V_{bd}$ represents the built-in voltage at zero bias, $V_{bf}$ represents the bias required to separate the quasi-Fermi levels inside the active region (a quantity which is very well approximated by a constant value above the lasing threshold), and $\Delta V$ takes on the same meaning as in (1).

Upon substitution of (2) into (1), it is clear that the solution for the extent of heterobarrier lowering for a given forward current density $J$ must be found using iterative methods. However, for small $\Delta V$ relative to $V_{bd} - V_{bf}$, the rate of change of the right-hand side of (1) with $\Delta V$ is small compared to the rate of change of the left-hand side. It may be seen from Fig. 2 that, over a broad range of parameters, the solution for $\Delta V$ from (1) is approximated well by neglecting its influence on $V_1$.

As a further simplification, the behavior of (1) may be subdivided into two regimes: one which is linear in $J$ and another which is logarithmic in $J$. For current densities less than

$$J_{transition} = \frac{\sqrt{2} \mu_p N_A q V_1 kT}{L_D \exp \left( \frac{\Delta E_V - \Delta E_{FR} - \Delta E_{FR}}{kT} \right)}$$  (3)

We label this the resistive regime of the heterojunction $V - J$ characteristic. In (4), $J_{hij}$ is given by

$$\Delta V = R_{hij} J_{act}.$$  (4)

(1) may be approximated by

$$\Delta V = R_{hij} J_{act}.\quad (4)$$

We label this the resistive regime of the heterojunction $V - J$ characteristic. In (4), $R_{hij}$ is given by

$$R_{hij} = \frac{kT}{q} \frac{L_D \exp \left( \frac{\Delta E_V - \Delta E_{FR} - \Delta E_{FR}}{kT} \right)}{\sqrt{2} \mu_p N_A q V_1 kT}.$$  (5)
In comparison, for current densities substantially above the transition current density, the dependence of Fig. 2 corresponds to a diode-like regime of heterojunction behavior

\[ J_{\text{act}}(V) = J_{0} \exp \left( \frac{q\Delta V}{kT} \right) \]  

where

\[ J_{0} = \frac{\sqrt{2} \mu_p N_{AV} q V}{L_{D}} \exp \left( \frac{\Delta E_{V} - \Delta E_{Fp} - \Delta E_{Fp}}{kT} \right). \]  

In Fig. 3, we plot the exact value of \( \Delta V \) for reference and plot the approximations of (4) and (6).

Whether the heterojunction \( V-J \) characteristic takes on a resistive or a diode-like character determines the qualitative and quantitative picture of overbarrier leakage. Considering drift and diffusive leakage of electrons into the p-cladding layer, we write [7], [8]

\[ J_l = qD_{n}n_{\text{clad}} \left( \frac{1}{L_{n}} + \frac{1}{L_{n,f}} \frac{\cot\text{Li}}{\sqrt{1 + \frac{1}{L_{n,f}^2}}} + \frac{1}{L_{n,f}} \right). \]  

where \( x_p \) is the p-cladding layer thickness. In the limit of a p-contact, which is far in terms of minority carrier diffusion lengths from the active region, we may approximate (8) by

\[ J_l = qD_{n}n_{\text{clad}} \left( \frac{1}{L_{n,f}} \right). \]  

Using Boltzmann’s approximation, we may relate the leakage current out of the active region to the (known) carrier density within the active region

\[ J_l = qD_{n}n_{\text{act}} \left( \frac{1}{L_{n,f}} + \frac{1}{L_{n,f}^2} \right) \exp \left( \frac{-\Delta E_{\text{in,conf}} - \Delta V}{kT} \right). \]  

where \( \Delta E_{\text{in,conf}} \) is the initial confinement barrier at zero forward bias. We see that drift leakage is of the same magnitude as diffusive leakage when \( L_{n} = L_{n,f} \). We therefore define a transition current between diffusive dominated and drift dominated regimes

\[ J_{t,\text{transition}} = 2 \left( \frac{kT}{q} \right) \left( \frac{\sigma_p}{L_{n}} \right). \]  

If the current is below this value, then diffusion will dominate and we can simplify (10) by discarding the \( L_{n,f} \) influence

\[ J_{t,\text{diffusive}} = \frac{qD_{n}n_{\text{act}}}{L_{n}} \exp \left( \frac{-\Delta E_{\text{in,conf}} - \Delta V}{kT} \right). \]  

Above this transition current, drift will dominate:

\[ J_{t,\text{drift}} = \frac{2qD_{n}n_{\text{act}}}{L_{n,f}} \exp \left( \frac{-\Delta E_{\text{in,conf}} - \Delta V}{kT} \right). \]  

In Fig. 4, we plot the exact value of \( J_t \) for reference and plot the approximations (12) and (13).

We are now able to apply the approximation for \( \Delta V \) into the different regimes of the leakage model. In this particular case, we use the parameters for a laser operating at 1.55 \( \mu \)m at 350 K, also listed in Table I, which permits us to illustrate a case of great technological relevance. It is clear that one cannot use the resistive approximation for \( \Delta V \) in the case of drift leakage, nor can one use the diode-like diffusive approximation when \( J_{t,\text{transition}} < J_{t,\text{transition}} \).

Using (4) and (6) combined with (12) and (13), the four regimes can be expressed analytically as

\[ J_{l,\text{resistive,drift}} \propto \alpha \exp(\gamma J_{\text{act}}) \]
\[ J_{l,\text{resistive,drift}} \propto \beta J_{l,\text{act}} \exp(\gamma J_{\text{act}}) \]
\[ J_{l,\text{diode-like,drift}} \propto \gamma J_{l,\text{act}} \]
\[ J_{l,\text{diode-like,drift}} \propto \beta \gamma J_{l,\text{act}} \]

where \( \alpha, \beta, \) and \( \gamma \) are independent of \( J_{\text{act}} \) (do depend on \( T, N_{AV}, \mu_p \) etc.). In Fig. 5, we show the appropriate approximations when \( J_{t,\text{transition}} < J_{l,\text{transition}} \) using (14).
It is apparent that the qualitative dependencies on forward current density differ markedly in the resistive and diode-like regimes. This has important implications as to the anticipated qualitative dependence of the light–current characteristic. We can write

\[ P_{\text{CUT}} \propto J_{\text{act}} - J_{\text{th}} = J_{\text{tot}} - J_{\text{th}} - J_{\text{th}}. \] (15)

As illustrated in Fig. 1(b), the internal efficiency \( \eta \) may be written

\[ \frac{1}{\eta} = \frac{\partial J_{\text{tot}}}{\partial J_{\text{act}}} = 1 + \frac{\partial J_t}{\partial J_{\text{act}}}. \] (16)

By using (16) on the four regimes in (14), we find for the internal efficiencies

\[ \eta_{\text{resistive,diffusive}} = \frac{1}{1 + \alpha \gamma \exp(\gamma J_{\text{act}})} \]

\[ \eta_{\text{resistive,drift}} = \frac{1}{1 + \beta(1 + \gamma \beta) \exp(\gamma J_{\text{act}})} \]

\[ \eta_{\text{diode-like,diffusive}} = \frac{1}{1 + 2 \beta \gamma J_{\text{act}}} \]

\[ \eta_{\text{diode-like,drift}} = \frac{1}{1 + 2 \beta \gamma \beta J_{\text{act}}}. \] (17)

Knowing that typical threshold values are \( 1 \times 10^7 \) A/m², we show in Fig. 6 the internal efficiency as a function of current. The bold line is calculated using the exact values for \( \Delta V \) and (10). Using parameters for the 1.3-μm laser at 337 K, outlined in Table I, the transition currents are equal, \( J_{\text{transmission}} = J_{\text{transition}} = 2.3 \times 10^7 \) A/m², just above threshold. This indicates that we should use the resistive/diffusive- and diode-like/drift approximations, the first and fourth equation from (17), respectively. Because the transition current is almost the same as the threshold current, the approximation for currents higher than threshold is well approximated by the diode-like/drift approximation, as can be seen in Fig. 6.

### III. Consequences

In this section, we use our model to explore the effects of several physical laser parameters on the internal quantum efficiency within two devices, one designed for operation at 1.3 μm, the other at 1.55 μm. Both are assumed to be built in the InGaAsP-InP material system. We begin by using values from Table I and varying the temperature. Fig. 7 shows the efficiencies for both lasers at 300 and 400 K.

The results of Figs. 7 and 8 are for internal quantum efficiency, i.e., the net efficiency with which carriers are injected into the laser active region and participate in stimulated recombination. The external quantum efficiency of 1300-nm lasers is in fact typically higher than that of 1550-nm devices, predominantly a consequence of weaker inter-valence band absorption (IVBA) at shorter wavelengths [9].
doping profile of highly diffusive zinc which results from epitaxial growth. Zn is also known to rearrange within device lifetimes, as a result, caution is usually exercised in positioning the Zn doping front. It is therefore possible for the p-doping level over tens of nanometers, which define the p-cladding-to-active-region interface, to be much lower than the peak or average level throughout the InP p-contact. The 1.3-µm laser shows serious deleterious effects if the doping level is below $2 \times 10^{17} \text{ cm}^{-3}$. Even in the much better confined 1.55-µm laser, the internal efficiency is merely 0.9 at $2 \times 10^{8} \text{ A/m}^2$. A 1.55-µm device operating with significant active region heating and modest doping at the depletion layer of $2 \times 10^{17} \text{ cm}^{-3}$ may thus suffer from severe degradation caused by overbarrier leakage.

IV. Conclusions

We have made a model for overbarrier leakage by approximating the behavior of a coupled self-consistent system over various regimes. We defined a transition current which separates two different regimes in the $V-J$ heterojunction characteristic and treated the effects of drift and diffusive leakage of carriers through the depletion layer.

We have shown that, by using this analytical physical model for overbarrier leakage in a semiconductor laser, we can immediately see the influence of key parameters on lasing efficiency. We have shown that, in worst case scenarios, high temperatures and low doping levels, a 1.55-µm laser can exhibit a dramatic performance degradation as a result of overbarrier leakage.

Acknowledgment

The authors would like to thank J. K. Snel, E. C. F. Wong, and F. Chang for the many technical discussions and support.

References

Igor M. P. Aarts was born on February 26, 1976. He is currently working toward the M.S. degree in applied physics at the Eindhoven University of Technology, Eindhoven, The Netherlands. As an exchange student, he visited the group of Prof. Sargent at the University of Toronto. His supervisor at the Eindhoven University of Technology is Prof. J. H. Wolter, the scientific director of COBRA, the Inter-University Research Institute on Communication Technology Basic Research and Applications.

Edward H. Sargent holds the Nortel Junior Chair in Emerging Technologies in the Department of Electrical and Computer Engineering at the University of Toronto, Toronto, ON, Canada. He leads a group of 12 graduate and post-doctoral researchers in the areas of semiconductor quantum electronic devices, photonic crystal applications, hybrid organic–inorganic quantum dot electroluminescence, and multiple-access optical networks.

Prof. Sargent was awarded the Silver Medal of the Natural Sciences and Engineering Research Council of Canada in 1999 for his work on the lateral current injection lasers. Also, in 1999, he won the Premier’s Research Excellence Award in recognition of research into the application of photonic crystals in lightwave systems.