Optical limiting and intensity-dependent diffraction from low-contrast nonlinear periodic media: Coupled-mode analysis

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The method of multiple scales is used to develop a procedure for obtaining coupled-mode equations in low-contrast nonlinear photonic crystals periodic in one, two, or three dimensions. Coupled-mode equations for three coupled modes in a two-dimensional (2D) hexagonal lattice are obtained in this way and solved numerically. We show that 2D low-contrast nonlinear photonic crystals support optical limiting and intensity-dependent diffraction.

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I. INTRODUCTION

Nonlinear photonic crystals may enable ultrafast all-optical signal processing. One-dimensionally periodic nonlinear media exhibit switching [1–5], pulse compression [6,7], and limiting [8,9]. Analysis of these devices is facilitated by development of one-dimensional (1D) nonlinear coupled mode equations (CME's) [8,10].

We develop herein, using the method of multiple scales, CME's for low-contrast Kerr-nonlinear photonic crystals in one, two, or three dimensions. By low contrast we intend that the variation in the linear index of refraction be small compared to the average index. We place our emphasis on this class of media because proposals for devices based on it show great promise [5,8,10,11] and because a recent experimental realization [12] and characterization of such a device suggests the need for further theoretical development.

The CME's derived in this paper allow us to analyze the intensity-dependent behavior of multidimensionally periodic low-contrast nonlinear media. Intensity-dependent diffraction in 1D structures underlies phenomena such as optical bistability [13,14], which in turn give rise to the functions listed above [15]. The present equations reduce as special cases to those used in the analysis of 1D periodic nonlinear media [8,16]. The applicability of the CME's to multidimensional media will facilitate the extension of previous work in 1D devices to higher dimensions.

The CME's presented here can describe resonant light incident from any direction on a medium having any periodic index of refraction profile. This is in contrast to some previous works wherein the index profile was fixed [17–20]. Because it is explicitly intended for low-contrast photonic crystals, the method presented here may provide a more direct means for their analysis than the alternative of considering high-contrast methods [21] in the limit of low index contrast.

Analysis of nonlinear periodic media often proceeds using perturbative approaches [22–25]. These methods treat the nonlinearity, and possibly the periodicity, as perturbations to a medium in which solutions to Maxwell's equations are known. In this way, the normal modes of these unperturbed media, and in particular how they interact and evolve under the perturbation, become the focus of the study of nonlinear periodic media.

The presence of a perturbation, no matter how weak, precludes independence in the evolution of the normal modes of the unperturbed medium. However, for a sufficiently weak nonlinearity, there will exist two distinct scales in space and time, one proper to the unperturbed normal modes of the linear photonic crystal, the other to the distances and times over which they interact because of the perturbation [26]. The evolution of an electromagnetic field within a nonlinear periodic medium is thus conveniently expressed in terms of normal modes modulated by slowly varying envelopes. Constructing the fields in this manner is central to the method of multiple scales, which is the basis for our analysis.

The method of multiple scales is a commonly used perturbative technique [24,27,28]. It has been deployed previously in, for example, deep nonlinear one-dimensional gratings [22,29] in which the envelope modulating a Bloch mode of the periodic unperturbed medium was found to satisfy a nonlinear Schrödinger equation. More recently, the technique has been employed in studying high-contrast nonlinear photonic crystals [23,30]. The equations governing the envelopes were parametrized by features relating to the unperturbed medium and, in particular, its band structure. These included features such as group velocity and group velocity dispersion.

In the present work, we consider instead multidimensionally periodic media whose built-in linear contrast is small compared to their average index. The nonlinear component of their refractive index modulation is necessarily small as well as a result of the empirical weakness of nonlinear optical response. We explore intensity-dependent transmission in certain systems, predicting optical limiting, and find conditions for intensity-dependent diffraction of an incident beam.

II. DERIVATION

We consider isotropic media possessing small, periodic variations in both the linear and nonlinear components of their indices of refraction. It is by treating the small variation in the index of the structure as a perturbation to a homogeneous linear medium that we obtain an approximate solution to Maxwell's equations.

We begin with the wave equation for the electric field inside a medium perturbed by a position-dependent nonlinear index of refraction:

$$\nabla^2\mathbf{E} - \nabla(\nabla\cdot\mathbf{E}) = \frac{n^2(\mathbf{x},|\mathbf{E}|^2)}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}, \qquad (1)$$

which has assumed

$$\left\| n^2(\mathbf{x},|\mathbf{E}|^2)\frac{\partial^2\mathbf{E}}{\partial t^2}\right\| \gg \left\| 2\frac{\partial\mathbf{E}}{\partial t}\frac{\partial n^2}{\partial t} + \mathbf{E}\frac{\partial^2 n^2}{\partial t^2}\right\|.$$

Inside the perturbed medium, the index of refraction is written as

$$n(\mathbf{x},|\mathbf{E}|^2) = n_0 + \xi\Delta^l(\mathbf{x}) + \xi|\mathbf{E}|^2\Delta^{nl}(\mathbf{x}), \qquad (2)$$

where n_0 is the index of the linear, homogeneous, unperturbed medium and ξ is a perturbation parameter characterizing the strength of the index perturbation. The terms $\Delta^l(\mathbf{x})$ and $\Delta^{nl}(\mathbf{x})$, both periodic, are the linear and nonlinear components of the perturbation. The effects of the Kerr nonlinearity in the perturbed medium are manifest in the term $|\mathbf{E}|^2\Delta^{nl}(\mathbf{x})$.

The relevant quantity in the wave equation (1) is the square of the total index which is given by

$$n^2(\mathbf{x},|\mathbf{E}|^2) = n_0^2 + 2n_0\xi[\Delta^l(\mathbf{x}) + \Delta^{nl}(\mathbf{x})|\mathbf{E}|^2]. \qquad (3)$$

Because our analysis will terminate at an $O(1)$ approximation of the electric field, the $O(\xi^2)$ term in Eq. (3) has been omitted.

The perturbation terms $\Delta^l(\mathbf{x})$ and $\Delta^{nl}(\mathbf{x})$ can be written as Fourier series using reciprocal lattice vectors:

$$\Delta^l(\mathbf{x}) = \sum_{all\mathbf{G}} \Delta^l_{\mathbf{G}} e^{-i(\mathbf{G}\cdot\mathbf{x})},$$

$$\Delta^{nl}(\mathbf{x}) = \sum_{all\mathbf{G}} \Delta^{nl}_{\mathbf{G}} e^{-i(\mathbf{G}\cdot\mathbf{x})}. \qquad (4)$$

The solutions of the wave equation (1) are parameterized through ξ. Furthermore, when $\xi=0$, those solutions are known to be plane waves, given that the unperturbed medium is charge free. Accordingly, the field is expanded in an asymptotic series about $\xi=0$,

$$\mathbf{E}(\mathbf{r},t) = \sum_{m=0}^{\infty} \mathbf{E}_m(\mathbf{x},t)\xi^m, \qquad (5)$$

from which an approximation to the field can be obtained by truncating the series at a sufficiently high order.

The perturbation parameter ξ is additionally used in the definition of new time and space scales, $\mathbf{X}_F=\mathbf{x}$, $\mathbf{X}_S=\xi\mathbf{x}$, $T_F=t$, and $T_S=\xi t$ where F and S, respectively, denote fast and slow. These scales will be considered as being independent, most importantly when differentiating with respect to $\mathbf{x}$ or t in Eq. (1). The derivatives relevant to the wave equation (1) become

$$\frac{\partial^2}{\partial x_i^2} = \frac{\partial^2}{\partial X_{F,i}^2} + 2\xi\frac{\partial^2}{\partial X_{F,i}\partial X_{S,i}} + \xi^2\frac{\partial^2}{\partial X_{S,i}^2}$$

$$\frac{\partial^2}{\partial x_i\partial x_j} = \frac{\partial^2}{\partial X_{F,i}\partial X_{F,j}} + \xi\left(\frac{\partial^2}{\partial X_{S,i}\partial X_{F,j}} + \frac{\partial^2}{\partial X_{F,i}\partial X_{S,j}}\right) + \xi^2\frac{\partial^2}{\partial X_{S,i}\partial X_{S,j}}$$

$$\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial T_F^2} + 2\xi\frac{\partial^2}{\partial T_S\partial T_F} + \xi^2\frac{\partial^2}{\partial T_F^2}. \qquad (6)$$

The scale $\mathbf{X}_F$ is that of the periodic index perturbations, so that the series representing the perturbations (4) retain the same form but with $\mathbf{x}$ replaced by $\mathbf{X}_F$.

Equipped with the expansion (5) and the new derivatives (6), a recurrence equation for the $\mathbf{E}_m(\mathbf{X}_\beta,T_\beta)$ can be obtained by substituting the expansions into the wave equation (1) and then using the linear independence of the powers of ξ.

Beginning by collecting terms proportional to ξ^0 in Eq. (1), it is found that, to first order, the recurrence equation is the wave equation in the unperturbed medium:

$$\nabla_F^2\mathbf{E}_0 - \nabla_F(\nabla_F\cdot\mathbf{E}_0) = \frac{n_0^2}{c^2}\frac{\partial^2\mathbf{E}_0}{\partial T_F^2}, \qquad (7)$$

where $\nabla_F=\Sigma_{n=1}^3\mathbf{e}_n\partial/\partial X_{F,n}$. Here ∇_F^2, $\nabla_F\cdot$ are defined similarly.

Equation (7) differs slightly from the wave equation expected in a charge-free homogeneous medium because of the presence of the $\nabla_F(\nabla_F\cdot\mathbf{E}_0)$ term. However, taking into account another of Maxwell's equations, it will now be shown that that term is identically zero. Using the expansion (5), the divergence of the electric field is given by

$$\nabla\cdot\mathbf{E} = \nabla_F\cdot\mathbf{E}_0 + \xi\nabla_S\cdot\mathbf{E}_0 + \nabla\cdot\left(\sum_{m=1}^{\infty}\mathbf{E}_m\xi^m\right),$$

where the expansion $\nabla\cdot=\nabla_F\cdot+\xi\nabla_S\cdot$ has been used in the last line above; with $\nabla_F\cdot=\Sigma_{n=1}^3\partial/\partial X_{F,n}$ and $\nabla_S\cdot$ is defined similarly.

Now, invoking the constitutive relation $\mathbf{D}=n^2(\mathbf{r},|\mathbf{E}^2|)\mathbf{E}$ and noting that the absence of charge in the medium requires $\nabla\cdot\mathbf{D}=0$, we find that

$$\nabla\cdot n^2\mathbf{E} = 0$$

$$= 2n(\nabla n)\dot{E} + n^2\nabla\cdot\mathbf{E} = 2\xi n(\nabla_F n)\cdot\mathbf{E} + n^2\left[\nabla_F\cdot\mathbf{E}_0 + \xi\nabla_S\cdot\mathbf{E}_0 + \nabla\cdot\left(\sum_{m=1}^{\infty}\mathbf{E}_m\xi^m\right)\right]. \qquad (8)$$

In light of Eq. (8), it follows that $\nabla_F\cdot\mathbf{E}=0$ identically. The third line above follows from the index being dependent only on the fast scale $\mathbf{X}_F$, so that $\nabla_S n=0$.

Applying this result to Eq. (7) yields the following wave equation for $\mathbf{E}_0$:

$$\nabla_F^2\mathbf{E}_0 = \frac{n_0^2}{c^2}\frac{\partial^2\mathbf{E}_0}{\partial T_F^2}, \qquad (9)$$

the general solution to which is a superposition of plane waves $\mathbf{e}_{\mathbf{k},\lambda,}e^{i(\mathbf{k}\cdot\mathbf{X_F}-\omega T_F)}$ where ω and $\mathbf{k}$ satisfy $\|\mathbf{k}\|^2=\omega^2 n_0^2/c^2$.

Here $\mathbf{e}_{\mathbf{k},\lambda}$ denotes one of two unit vectors labeled by λ, which, together with $\mathbf{k}$, form an orthogonal triad.

For $\mathbf{E}_0$ monochromatic with frequency ω_0, it may be written as a sum of plane waves whose wave vectors lie on the sphere in $\mathbf{k}$ space centered at the origin with radius $n_0\omega_0/c$. We denote this sphere by (S), so that

$$\mathbf{E}_0(\mathbf{X}_\beta,T_\beta)=\sum_{\mathbf{k}}^{(S)}\mathbf{A}_\mathbf{k}(\mathbf{X}_S,T_S)\varphi_\mathbf{k}. \tag{10}$$

Here, $\varphi_\mathbf{k}$ denotes the plane wave with wave vector $\mathbf{k}$, $e^{i(\mathbf{k}\cdot\mathbf{X_F}-\omega_0 t)}$, and β denotes the dependence of $\mathbf{E}_0$ on all of the scales. The coefficients $\mathbf{A}_\mathbf{k}(\mathbf{X}_S,T_S)$, which depend only on the slow scales, are the envelopes governing the evolution of the normal modes under the perturbation. It is these envelopes that we intend to study through the CME's. We note that the transversality of the unperturbed normal modes requires that $\mathbf{A}_\mathbf{k}\cdot\mathbf{k}=0$.

Having obtained the leading order term, we now obtain $\mathbf{E}_1$ by collecting in the wave equation (1) all terms proportional to ξ and requiring that the resulting coefficient of ξ vanish. Required in the resulting equation is an expression for the intensity of the leading order term in terms of the mode envelopes. The intensity of the leading order term is given by

$$|\mathbf{E}_0|^2=\mathbf{E}_0\overline{\mathbf{E}}_0=\sum_{\mathbf{k}',\mathbf{k}''}^{(S)}\mathbf{A}_{\mathbf{k}'}\cdot\overline{\mathbf{A}_{\mathbf{k}''}}e^{i(\mathbf{k}'-\mathbf{k}'')\cdot\mathbf{X}_F},$$

since $\varphi_{\mathbf{k}'}\overline{\varphi_{\mathbf{k}''}}=e^{i(\mathbf{k}'-\mathbf{k}'')\cdot\mathbf{X}_F}$. Using this expression for the leading order intensity as well as the Fourier series for $\Delta^l(\mathbf{X}_F)$ and $\Delta^{nl}(\mathbf{X}_F)$, Eq. (4), and noting that $e^{-i\mathbf{G}\cdot\mathbf{X}_F}\varphi_\mathbf{k}=\varphi_{\mathbf{k}-\mathbf{G}}$, the $O(\xi)$ equation becomes

$$\left(\nabla_F^2\mathbf{E}_1-\nabla_F(\nabla_F\cdot\mathbf{E}_1)-\frac{n_0^2}{c^2}\frac{\partial^2\mathbf{E}_1}{\partial T_F^2}\right)_i=\sum_{\mathbf{k}}^{(S)}\Bigg[-i\left(\mathbf{k}\cdot\nabla_S A_{\mathbf{k},i}-\frac{1}{2}\mathbf{k}\cdot\frac{\partial\mathbf{A}_\mathbf{k}}{\partial X_{S,i}}-\frac{1}{2}k_i\nabla_S\cdot\mathbf{A}_\mathbf{k}+\frac{n_0^2\omega_0}{c^2}\frac{\partial A_{\mathbf{k},i}}{\partial T_S}\right)\varphi_\mathbf{k}-\frac{n_0\omega_0^2}{c^2}\sum_{\mathbf{G}}\Delta_\mathbf{G}^l A_{\mathbf{k},i}\varphi_{\mathbf{k}-\mathbf{G}}$$
$$-\frac{n_0\omega_0^2}{c^2}\sum_{\mathbf{k}',\mathbf{k}''}^{(S)}\sum_{\mathbf{G}}\Delta_\mathbf{G}^{nl}(\mathbf{A}_{\mathbf{k}'}\cdot\overline{\mathbf{A}}_{\mathbf{k}''})A_{\mathbf{k},i}\varphi_{\mathbf{k}+\mathbf{k}'-\mathbf{k}''-\mathbf{G}}\Bigg]. \tag{11}$$

It is from Eq. (11) that we obtain the coupled mode equations. If Eq. (11) is cast into the form $L\mathbf{E}_1=\mathbf{f}$, where $L=\nabla_F^2-\nabla_F(\nabla_F\cdot)-(n_0^2/c^2)\partial^2/\partial T_F^2$ is a self adjoint linear operator and $\mathbf{f}$ a vector whose ith component is given by the right side of Eq. (11), then for the solution $\mathbf{E}_1$ to exist, $\mathbf{f}$ must be in the range of L, denoted $R(L)$. This in turn requires that $\mathbf{f}$ be orthogonal to the null space of the adjoint of L, which is simply the null space of L, $N(L)$. Here $N(L)$ contains the solutions to the homogeneous problem $L\mathbf{E}_1=0$. In particular, $N(L)\supset\{\mathbf{e}_\mathbf{k}\varphi_\mathbf{k}:\|\mathbf{k}\|=n_0\omega_0/c,\mathbf{e}_\mathbf{k}\cdot\mathbf{k}=0\}$. Thus, $\langle\mathbf{e}_\mathbf{k}\varphi_\mathbf{k},\mathbf{f}\rangle=\Sigma_{i=1}^3 e_i\langle\varphi_\mathbf{k},f_i\rangle=0$ for all $\varphi_\mathbf{k}$ such that $\|\mathbf{k}\|=n_0\omega_0/c$. Writing this out explicitly results in the following equation:

$$i\left(\mathbf{k}\cdot\nabla_S(\mathbf{e}_\mathbf{k}\cdot A_\mathbf{k})-\frac{1}{2}\mathbf{k}\cdot(\mathbf{e}_\mathbf{k}\cdot\nabla_S)\mathbf{A}_\mathbf{k}+\frac{n_0^2\omega_0}{c^2}\frac{\partial(\mathbf{e}_\mathbf{k}\cdot A_\mathbf{k})}{\partial T_S}\right)\langle\varphi_\mathbf{k},\varphi_\mathbf{k}\rangle-\frac{n_0\omega_0^2}{c^2}\sum_{\mathbf{k}'}^{(S)}\sum_{all\mathbf{G}}\Delta_\mathbf{G}^l(\mathbf{e}_\mathbf{k}\cdot\mathbf{A}_{\mathbf{k}'})\langle\varphi_\mathbf{k},\varphi_{\mathbf{k}'-\mathbf{G}}\rangle$$
$$-\frac{n_0\omega_0^2}{c^2}\sum_{\mathbf{k}',\mathbf{k}'',\mathbf{k}'''}^{(S)}\sum_{all\mathbf{G}}\Delta_\mathbf{G}^{nl}(\mathbf{A}_{\mathbf{k}''}\cdot\overline{\mathbf{A}}_{\mathbf{k}'''})(\mathbf{e}_\mathbf{k}\cdot\mathbf{A}_{\mathbf{k}'})\langle\varphi_\mathbf{k},\varphi_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}'''-\mathbf{G}}\rangle=0, \tag{12}$$

where the notation $(\mathbf{e}_\mathbf{k}\cdot\nabla_S)\mathbf{A}_\mathbf{k}$ refers to a vector whose jth component is $\Sigma_{i=1}^3 e_{\mathbf{k},i}(\partial/\partial X_{S,i})A_{\mathbf{k},j}$. This set of equations, for $\mathbf{k}$ ranging over S and for any $\mathbf{e}_\mathbf{k}$ such that $\mathbf{e}_\mathbf{k}\cdot\mathbf{k}=0$, is the set of CME's for the medium at the frequency ω_0.

While this projection can be carried out for any $\mathbf{e}_\mathbf{k}$ satisfying $\mathbf{e}_\mathbf{k}\cdot\mathbf{k}=0$, only two linearly independent equations will result. In particular, all of the equations obtained in this way are linearly dependent on the two equations that result from projecting Eq. (11) onto two linearly independent vectors $\mathbf{e}_{\mathbf{k},1}$ and $\mathbf{e}_{\mathbf{k},2}$ for each $\mathbf{k}$. This follows from the linearity of Eq. (12) with respect to $\mathbf{e}_\mathbf{k}$ and guarantees that of all the projections $\langle\mathbf{e}_\mathbf{k}\varphi_\mathbf{k},\mathbf{f}\rangle$ vanish. Thus, given the condition $\mathbf{k}\cdot\mathbf{A}_\mathbf{k}=0$ as well as these two linearly independent differential equations, we find that there exist three independent equations for each mode envelope $\mathbf{A}_\mathbf{k}$. That is, there exists one equation for each unknown, or each component of the vector $\mathbf{A}_\mathbf{k}$.

Writing Eq. (12) in its final form requires finding all of the nonzero projections $\langle\varphi_\mathbf{k},\varphi_{\mathbf{k}'-\mathbf{G}}\rangle$ and $\langle\varphi_\mathbf{k},\varphi_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}'''-\mathbf{G}}\rangle$. From the orthogonality of the normal modes, this is equivalent to finding all wave vectors $\mathbf{k}'$ and all reciprocal lattice vectors $\mathbf{G}$ such that $\mathbf{k}'-\mathbf{G}=\mathbf{k}$ as well as all $\mathbf{k}'$, $\mathbf{k}''$, $\mathbf{k}'''$, and $\mathbf{G}$ such that $\mathbf{k}'+\mathbf{k}''-\mathbf{k}'''-\mathbf{G}=\mathbf{k}$. The modes corresponding to those wave vectors are then said to be coupled because, as will be shown shortly, the envelopes of these modes influence the evolution of one another.

Turning now to the problem of finding the nonzero terms $\langle\varphi_\mathbf{k},\varphi_{\mathbf{k}-\mathbf{G}}\rangle$, we first consider the mode coupling caused by the linear index perturbation. As is evident in Eq. (11), it is the linear component of the index perturbation that couples modes $\varphi_\mathbf{k}$, $\varphi_{\mathbf{k}'}$ corresponding to wave vectors that satisfy $\mathbf{k}$

$=\mathbf{k}'-\mathbf{G}$ and $\|\mathbf{k}\|=\|\mathbf{k}'\|=n_0\omega_0/c$. Now, the condition $\|\mathbf{k}\|=\|\mathbf{k}'\|$ is equivalent to $\|\mathbf{k}\|^2=\|\mathbf{k}+\mathbf{G}\|^2$, which requires that $\mathbf{k}\cdot\mathbf{G}=-\|\mathbf{G}\|/2$. By the construction of Brillouin zones, the wave vector $\mathbf{k}$ must then necessarily lie on the face of a particular Brillouin zone and, in particular, the face corresponding to the reciprocal lattice vector $\mathbf{G}$. Conversely, for $\mathbf{k}$ to lie on the face of a Brillouin zone, it is sufficient for $\mathbf{k}\cdot\mathbf{G}=-\|\mathbf{G}\|/2$ for some reciprocal lattice vector $\mathbf{G}$, in which case $\|\mathbf{k}+\mathbf{G}\|=\|\mathbf{k}\|$ and $\mathbf{k}$ is then coupled to $\mathbf{k}+\mathbf{G}$. So, for the wave vector $\mathbf{k}$ to be coupled to another wave vector lying on the sphere S, it is necessary and sufficient for $\mathbf{k}$ to lie on the face of a Brillouin zone of the index lattice. A wave vector is coupled to more than one other wave vector when it lies on more than one face of a Brillouin zone. This occurs when the wave vector lies at the intersection of some number of faces of a Brillouin zone. This number is then the number of directions to which the given wave vector is coupled. It is coupled by the reciprocal lattice vectors corresponding to the faces on which it lies.

This process of identifying the modes coupled by the linear perturbation to a given mode $\varphi_{\mathbf{k}}$ is exhaustive in that all of the wave vector $\mathbf{k}'$ such that $\mathbf{k}=\mathbf{k}'-\mathbf{G}$ with $\|\mathbf{k}'\|=\|\mathbf{k}\|$ can be determined from the position of $\mathbf{k}$ with respect to the Brillouin zones of the index lattice. Moreover, modes in the set of modes that a given mode is coupled to are themselves coupled only within that set. To see this, suppose that a wave vector $\mathbf{k}_1$ was coupled to another $\mathbf{k}_2$ and that $\mathbf{k}_2$ was itself coupled to $\mathbf{k}_3$. This requires that $\|\mathbf{k}_1\|=\|\mathbf{k}_2\|$ and $\|\mathbf{k}_2\|=\|\mathbf{k}_3\|$ as well as the existence of reciprocal lattice vectors $\mathbf{G}_{12}$ and $\mathbf{G}_{23}$ such that $\mathbf{k}_2=\mathbf{k}_1-\mathbf{G}_{12}$ and $\mathbf{k}_3=\mathbf{k}_2-\mathbf{G}_{23}$. This in turn implies that $\|\mathbf{k}_1\|=\|\mathbf{k}_3\|$ and that $\mathbf{k}_3=\mathbf{k}_1-(\mathbf{G}_{12}+\mathbf{G}_{23})$ so that $\mathbf{k}_1$ and $\mathbf{k}_3$ are indeed coupled inasmuch as $\mathbf{G}_{12}-\mathbf{G}_{23}$ is itself a reciprocal lattice vector.

Having characterized the modes coupled by the linear index perturbation, we now turn to the nonlinear perturbation, Δ^{nl}. The nonlinear component of the index perturbation couples modes whose wave vectors lie on the sphere S and that satisfy $\mathbf{k}'+\mathbf{k}''-\mathbf{k}'''-\mathbf{G}=\mathbf{k}$ for some reciprocal lattice vector $\mathbf{G}$. Now, all of the wave vectors that are coupled by the linear perturbation and that are characterized in the preceding paragraph necessarily differ by a reciprocal lattice vector. The condition for coupling by the nonlinear perturbation can be rewritten as $(\mathbf{k}-\mathbf{k}''')+(\mathbf{k}'-\mathbf{k}'')=\mathbf{G}$. So, if those wave vectors are ones coupled by the linear perturbation, there always exists a reciprocal lattice vector $\mathbf{G}$ satisfying that condition, insofar as $\mathbf{k}-\mathbf{k}'''$ and $\mathbf{k}'-\mathbf{k}''$ are both reciprocal lattice vectors in this case. It is sufficient, then, for modes to be coupled by the linear perturbation for them to be coupled by the nonlinear perturbation. The converse, however, is not true. It is not necessary for modes to be coupled by the linear perturbation to be coupled by the nonlinear perturbation. This can be seen from the condition $(\mathbf{k}-\mathbf{k}''')+(\mathbf{k}'-\mathbf{k}'')=\mathbf{G}$ when $\mathbf{k}$ and $\mathbf{k}'''$ are linearly coupled and when $\mathbf{k}'=\mathbf{k}''$. In this case, $\mathbf{k}'$ and so $\mathbf{k}''$ can be any wave vectors on the sphere S and still satisfy the nonlinear coupling condition. In particular, the nonlinear perturbation couples together all of the modes in the expansion (5), a difficulty that we will address shortly.

Having determined how to identify the nonzero projections in Eq. (12), it can be put into its final form by replacing each of the double sums therein by the terms corresponding to those nonzero projections.

The effect of the nonlinearity coupling all modes together is to introduce an infinite number of terms into Eq. (12) via the projections $\langle\varphi_{\mathbf{k}},\varphi_{\mathbf{k}+\mathbf{k}''-\mathbf{k}'''-\mathbf{G}}\rangle$. Also, because there exists a solvability condition for each wave vector lying on the sphere S, there exists an infinite number of coupled mode equations. However, only a finite number of equations, those corresponding to the modes that are coupled by the linear perturbation, contain terms from both the linear and nonlinear perturbations. If the leading order solution $\mathbf{E}_0$ were to be a sum only of those modes which are coupled by the linear perturbation, the number of coupled mode equations would become finite and the equations themselves would contain only a finite number of terms.

Thus, the approach we adopt in finding the CME is, given a wave vector $\mathbf{k_i}$ that is known to be present in the medium, to expand the leading order solution $\mathbf{E}_0$ only in terms of $\varphi_{\mathbf{k}_i}$ and the modes to which it is coupled. To solve for the mode envelopes $\mathbf{A}_{\mathbf{k}}$, we use the projection (12) with two linearly independent solutions for each wave vector, $\mathbf{e}_{\mathbf{k},m}\varphi_{\mathbf{k}}$, $m=1, 2$. These equations, in conjunction with the condition $\mathbf{A}_{\mathbf{k}}\cdot\mathbf{k}=0$, provide a means of determining the $\mathbf{A}_{\mathbf{k}}$.

III. APPLICATIONS

We now apply the preceding derivation to the investigation of the intensity-dependent behavior of two-dimensionally periodic nonlinear media. In particular, we study the coupling of two modes in a 2D lattice wherein the electric field is assumed to be orthogonal to the lattice plane.

We begin with a two-dimensional lattice taken as lying in the xy plane and having any lattice geometry. A mode $\varphi_{\mathbf{k}}$ whose wave vector $\mathbf{k}$ lies on an edge of a Brillouin zone of this lattice, but not at the intersection of any edges, is coupled only to one other mode $\varphi_{\mathbf{k}'}$. The wave vector $\mathbf{k}'$ is related to $\mathbf{k}$ by $\mathbf{k}'=\mathbf{k}+\mathbf{G}$ where $\mathbf{G}$ is the reciprocal lattice vector corresponding to the Brillouin zone edge on which $\mathbf{k}$ lies. If this mode is assumed to be present in the medium, then, assuming that the leading order term $\mathbf{E}_0$ can be expressed as a sum of $\varphi_{\mathbf{k}}$ and the modes to which it is coupled by the linear perturbation,

$$\mathbf{E}_0(\mathbf{X}_\beta,T_\beta)=\mathbf{e_z}[A_1(\mathbf{X}_S,T_S)\varphi_{\mathbf{k}_1}+A_2(\mathbf{X}_S,T_S)\varphi_{\mathbf{k}_2}],$$

where is has been additionally assumed that the field is polarized perpendicular to the lattice plane. The polarization assumption allows for the use of scalar mode envelopes. Moreover, it requires projecting Eq. (12) only onto solutions $\mathbf{e_k}\varphi_{\mathbf{k}}$ for which $\mathbf{e_k}=\mathbf{e_z}$.

What results from projecting Eq. (12) onto $\mathbf{e_z}\varphi_{\mathbf{k}}$ in this case is the CME for A_1:

$$-i\left(\mathbf{k}_1\cdot\nabla_S A_1+\frac{n_0^2\omega_0}{c^2}\frac{\partial A_1}{\partial T_S}\right)+\frac{n_0\omega_0^2}{c^2}\Delta^l_{\mathbf{G}}A_2+\frac{n_0\omega_0^2}{c^2}\times\{\Delta_0^{nl}A_1(|A_1|^2+2|A_2|^2)+\Delta^{nl}_{\mathbf{G}}[A_1^2\overline{A_2}+A_2(2|A_1|^2+|A_2|^2)]+\Delta^{nl}_{2\mathbf{G}}\overline{A_1}A_2^2\}=0.$$

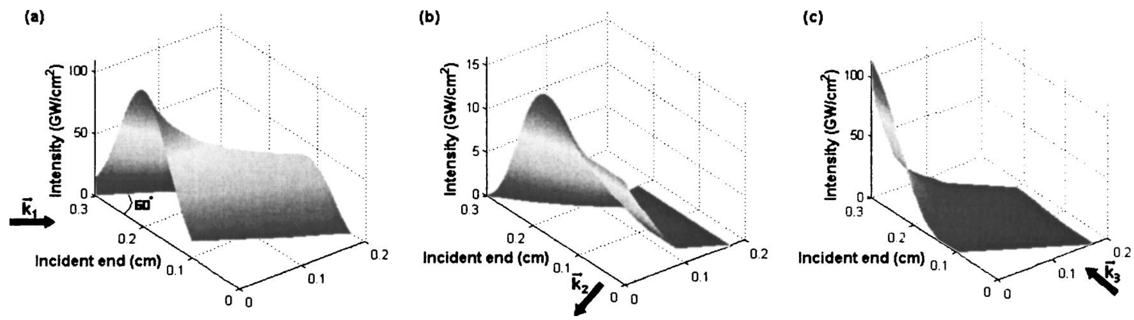

FIG. 1. Intensities of the modes with wave vectors (a) $\mathbf{k}_1$ (b) $\mathbf{k}_2$, and (c) $\mathbf{k}_3$ over the two-dimensional crystal when $I_{peak}=110\ \mathrm{GW/cm^2}$. Here $\mathbf{k}_1$ is the wave vector of the incident beam.

Similarly, projecting onto $\mathbf{e_z}\varphi_{\mathbf{k}'}$ and requiring that the terms vanish results in the second coupled-mode equation

$$-i\left(\mathbf{k}_2\cdot\nabla_S A_2+\frac{n_0^2\omega_0}{c^2}\frac{\partial A_2}{\partial T_S}\right)+\frac{n_0\omega_0^2}{c^2}\Delta^l_{\mathbf{G}}A_1+\frac{n_0\omega_0^2}{c^2}\{\Delta_0^{nl}A_2(2|A_1|^2+|A_2|^2)+\Delta^{nl}_{\mathbf{G}}[A_2^2\overline{A_1}+A_1(|A_1|^2+2|A_2|^2)]+\Delta^{nl}_{2\mathbf{G}}\overline{A_2}A_1^2\}=0. \tag{13}$$

It has been assumed here that the index perturbation $\Delta(\mathbf{X}_F)$ has inversion symmetry so that $\Delta(\mathbf{X}_F)=\Delta(-\mathbf{X}_F)$. This requires that its Fourier components satisfy $\Delta_{\mathbf{G}}=\Delta_{-\mathbf{G}}$. It has also been assumed that the component of the linear index perturbation that is constant in space is zero, so that $\Delta_0^l=0$. Apart from this and the assumption that the electric field is perpendicular to the lattice plane, the equations above are the coupled-mode equations for any two modes lying on a single edge of the Brillouin zone of the index lattice.

In the case that $\mathbf{k}=-\mathbf{G}/2$, for some reciprocal lattice vector $\mathbf{G}$, $\mathbf{k}$ necessarily lies on a single edge of the Brillouin zone of the index lattice and is only coupled by $\mathbf{G}$ to $\mathbf{k}+\mathbf{G}=\mathbf{G}/2=-\mathbf{k}$. Labeling the envelope for $\varphi_{\mathbf{k}}$ as A_+ and that for $\varphi_{-\mathbf{k}}$ as A_- and continuing to assume that the field is orthogonal to the lattice plane, the coupled-mode equations become

$$-i\left(\mathbf{k}\cdot\nabla_S A_+ +\frac{n_0^2\omega_0}{c^2}\frac{\partial A_+}{\partial T_S}\right)+\frac{n_0\omega_0^2}{c^2}\Delta^l_{\mathbf{G}}A_- +\frac{n_0\omega_0^2}{c^2}\times\{\Delta_0^{nl}A_-(|A_-|^2+2|A_+|^2)+\Delta^{nl}_{\mathbf{G}}[A_-^2\overline{A_+}+A_+(2|A_-|^2+|A_+|^2)]+\Delta^{nl}_{2\mathbf{G}}\overline{A_-}A_+^2\}=0,$$

$$-i\left(\mathbf{k}\cdot\nabla_S A_- +\frac{n_0^2\omega_0}{c^2}\frac{\partial A_-}{\partial T_S}\right)+\frac{n_0\omega_0^2}{c^2}\Delta^l_{\mathbf{G}}A_+ +\frac{n_0\omega_0^2}{c^2}\times\{\Delta_0^{nl}A_-(2|A_+|^2+|A_-|^2)+\Delta^{nl}_{\mathbf{G}}[A_-^2\overline{A_+}+A_+(|A_+|^2+2|A_-|^2)]+\Delta^{nl}_{2\mathbf{G}}\overline{A_-}A_+^2\}=0.$$

A new coordinate system can be defined such that its axes—say, X_S', Y_S', and Z_S'—are, respectively, along a unit vector in the direction of $\mathbf{k}$ and any two vectors that are orthonormal to $\mathbf{k}$ and to one another. The coordinates of a vector in the original coordinate system have the following relationships with X_S' of the new system:

$$\frac{\partial X_S}{\partial X_S'}=\hat{k}_x,\quad \frac{\partial Y_S}{\partial X_S'}=\hat{k}_y,\quad \frac{\partial Z_S}{\partial X_S'}=\hat{k}_z(=0),$$

where $\hat{\mathbf{k}}=(\hat{k}_x,\hat{k}_y,\hat{k}_z)$ is a unit vector in the direction of the wave vector $\mathbf{k}$.

Now, if for each envelope a new function $A_\lambda'(\mathbf{X}_S,t)$, $(\lambda=+,-)$ is defined such that $A_\lambda(\mathbf{X}_S,T_S)=A_\lambda'(\mathbf{T}^{-1}\mathbf{X}_S,T_S)$, where $\mathbf{T}$ is the change of basis matrix from the original coordinate system to the new one, then, using the chain rule,

$$\nabla_S A_+=\hat{k}_x\frac{\partial A_+}{\partial X_S}+\hat{k}_y\frac{\partial A_+}{\partial Y_S}+\hat{k}_z\frac{\partial A_+}{\partial Z_S}=\hat{k}_x\frac{\partial A_+}{\partial X_S}+\hat{k}_y\frac{\partial A_+}{\partial Y_S}=\frac{\partial A_+'}{\partial X_S'}.$$

Similarly, $\nabla_S A_-=\partial A_-'/\partial X_S'$.

So, finally, the equations above reduce, in the new coordinate system, to

$$-i\left(\frac{n_0\omega_0}{c}\frac{\partial A_+'}{\partial X_S'}+\frac{n_0^2\omega_0}{c^2}\frac{\partial A_+'}{\partial T_S}\right)+\frac{n_0\omega_0^2}{c^2}\Delta^l_{\mathbf{G}}A_-'+\frac{n_0\omega_0^2}{c^2}\times\{\Delta_0^{nl}A_-'(|A_-'|^2+2|A_+'|^2)+\Delta^{nl}_{\mathbf{G}}[A_-'^2\overline{A'}_+ +A_+'(2|A_-'|^2+|A_+'|^2)]+\Delta^{nl}_{2\mathbf{G}}\overline{A'}_-A_+'^2\}=0,$$

$$-i\left(-\frac{n_0\omega_0}{c}\frac{\partial A_+'}{\partial X_S'}+\frac{n_0^2\omega_0}{c^2}\frac{\partial A_-'}{\partial T_S}\right)+\frac{n_0\omega_0^2}{c^2}\Delta^l_{\mathbf{G}}A_+'+\frac{n_0\omega_0^2}{c^2}\times\{\Delta_0^{nl}A_-'(2|A_+'|^2+|A_-'|^2)+\Delta^{nl}_{\mathbf{G}}[A_-'^2\overline{A'}_+ +A_+'(|A_+'|^2+2|A_-'|^2)]+\Delta^{nl}_{2\mathbf{G}}\overline{A'}_-A_+'^2\}=0,$$

which are the familiar one-dimensional coupled-mode equations [8].

Thus, for any wave vector $\mathbf{k}$ lying in the plane of a 2D crystal such that $\mathbf{k}=-\mathbf{G}/2$ for some reciprocal lattice vector $\mathbf{G}$, the electric field can be expanded using modes whose amplitudes are determined by equations identical to those for a 1D crystal with a normally incident field. An important consequence of this is that the behavior of pulses incident on a 2D crystal from a direction that is parallel to some reciprocal lattice vector will be the same as that of a pulse incident on a one-dimensional crystal. Pulses in one-dimensional periodic Kerr-nonlinear structures are studied in [11].

When $\mathbf{k}=-\mathbf{G}/2$ for a reciprocal lattice vector $\mathbf{G}$, the incident field propagates in a direction normal to a family of lattice planes defined by $\mathbf{G}$. The set of lattice planes defined by $\mathbf{G}$ will in this case act as the boundaries between the alternating layers in a 1D crystals. The pulse will experience what is effectively a square wave index perturbation in one dimension with Fourier components Δ_0^{nl}, $\Delta_{\mathbf{G}}^{l}$, $\Delta_{\mathbf{G}}^{nl}$, and so on.

IV. SOLUTION FOR A HEXAGONAL-LATTICE PHOTONIC CRYSTAL

The case that we study in detail here is that of three coupled modes in a two-dimensional nonlinear, hexagonal photonic crystal throughout which the electric field is polarized perpendicularly to the plane of the crystal. We take this plane to be the xy plane. This choice of polarization permits the use of scalar envelopes in the zeroth-order correction to the electric field.

We take the hexagonal lattice to have lattice spacing a so that is has reciprocal lattice vectors $\mathbf{G}_{12}=(4\pi/\sqrt{3a})(1,0,0)$ and $\mathbf{G}_{13}=(4\pi/\sqrt{3a})\left(\frac{1}{2},\sqrt{3}/2,0\right)$. The $O(\xi^0)$ order correction to the field is taken to consist of the mode $\varphi_{\mathbf{k}_1}$ with wave vector $\mathbf{k}_1=(4\pi/\sqrt{3a})\left(\frac{1}{2},1/\sqrt{12},0\right)$ and those modes to which it is linearly coupled. From its position on the Brillouin zone of the index lattice, the modes to which $\varphi_{\mathbf{k}_1}$ is coupled are found to be those with wave vectors $\mathbf{k}_2=(4\pi/\sqrt{3a})\times\left(-\frac{1}{2},1/\sqrt{12},0\right)$ and $\mathbf{k}_2=(4\pi/\sqrt{3a})(0,1/\sqrt{3},0)$. Note that $\mathbf{k}_2=\mathbf{k}_1-\mathbf{G}_{12}$, $\mathbf{k}_3=\mathbf{k}_1-\mathbf{G}_{13}$, and $\mathbf{k}_3=\mathbf{k}_2-\mathbf{G}_{23}$ with $\mathbf{G}_{23}=\mathbf{G}_{12}-\mathbf{G}_{13}$.

So, if $\mathbf{E}_0=\mathbf{e}_{\mathbf{z}}\Sigma_{i=1}^{3}A_i\varphi_{\mathbf{k}_i}$, then using Eq. (12), the mode envelopes A_i must satisfy the following equations:

$$-i\frac{\pi}{a}\left(\frac{1}{2}\frac{\partial A_i}{\partial X_S}+\frac{1}{\sqrt{12}}\frac{\partial A_i}{\partial Y_S}+\frac{n_0^2\omega_0}{c}\frac{\partial A_i}{\partial T_S}\right)+\frac{n_0\omega_0^2}{c}(\Delta_{\mathbf{G}_{ij}}^{l}A_j+\Delta_{\mathbf{G}_{ik}}^{l}A_k)+\frac{n_0\omega_0^2}{c}(\Delta_0^{nl}A_i(|A_i|^2+2(|A_j|^2+|\Delta_{\mathbf{G}_{ij}}^{nl}\{A_i^2\overline{A_j}^2+A_j[|A_j|^2$$
$$+2(|A_i|^2+|A_k|^2)]\}+\Delta_{\mathbf{G}_{ik}}^{nl}\{A_i^2\overline{A_k}^2+A_k[|A_k|^2+2(|A_i|^2+|A_j|^2)]\}+2\Delta_{\mathbf{G}_{jk}}^{nl}A_i(A_j\overline{A_k}+\overline{A_j}A_k)+\Delta_{2\mathbf{G}_{ij}}^{nl}\overline{A_i}A_j^2+\Delta_{2\mathbf{G}_{ik}}^{nl}\overline{A_i}A_k^2$$
$$+\Delta_{\mathbf{G}_{ij}^{nl}+\mathbf{G}_{jk}}A_j^2\overline{A_k}+\Delta_{\mathbf{G}_{ik}^{nl}+\mathbf{G}_{jk}}\overline{A_j}A_k^2+2\Delta_{\mathbf{G}_{ij}+\mathbf{G}_{ik}}^{nl}\overline{A_i}A_jA_k)=0$$

for cyclic permutations of $(i,j,k)=(1,2,3)$. Note that the perturbations are assumed to be even and real so that $\Delta_{\mathbf{G}}=\Delta_{-\mathbf{G}}$.

Equations (14) were numerically solved for a specific crystal whose lattice consists of an array of hexagonally arranged cylindrical rods embedded in a filler matrix. The linear index of refraction was kept constant throughout the lattice, so that $\Delta^{l}(\mathbf{x})=0$ for all $\mathbf{x}$ and $n_0=1.5$. The Kerr-nonlinearity coefficient of the material comprising the rods was chosen as 4×10^{-5} $\mathrm{cm}^2/\mathrm{GW}$, while the nonlinear coefficient of the material surrounding the rods was the negative of this. The two-dimensional cross section of the crystal is a parallelogram of which both sides, oriented at 60° to one another, are 2 mm in length. The cross section consists of 800 rows and 800 slanted columns of rods. The radii of the rods are 0.93 μm, chosen so that the cross sectional areas of the rods equalled the area of the filler around them.

An iterative numerical method incorporating the use of finite differences was implemented to simulate a continuous wave incident on one side of the crystal. The wave has a Gaussian beam profile transverse to the incident side, given by $I_{in}(x)=I_{peak}e^{-200(x-0.01)^2}$ where x is the distance along the incident end of the crystal. The frequency of the input wave is 200 THz.

The steady state intensities of the three coupled waves over the two-dimensional lattice are plotted in Fig. 1 for the case of $I_{peak}=110$ $\mathrm{GW}/\mathrm{cm}^2$. The incident wave, which has the wave vector $\mathbf{k}_1$, is directed from the incident end to the end opposite the incident end. Figure 1(a) shows that the intensity of this wave is greatest where it enters the crystal and then diminishes in intensity as we move up the lattice in the direction of its wave vector, since the wave is diffracting into the other two modes $\varphi_{\mathbf{k}_2}$ and $\varphi_{\mathbf{k}_3}$. The intensities of these latter two waves are plotted in Figs. 1(b) and 1(c), respectively.

In Fig. 2, the power of each of the three modes exiting the crystal is plotted against the power of the input beam. The curve in Fig. 2(a) shows that the power of the light exiting the side opposite the incident end asymptotically approaches a limiting value as incident power is increased. Thus, this structure is an optical limiter. Figures 2(b) and 2(c) illustrate that as incident power increases, more and more power is diverted from the mode with wave vector $\mathbf{k}_1$ into the modes with wave vectors $\mathbf{k}_2$ and $\mathbf{k}_3$.

At low intensities, the nonlinear components of the refractive indices are very small, so the indices of the two different materials are nearly matched: incident light passes through the crystal with minimal scattering. As intensity is increased, the difference between the indices of the two materials grows, and more of the mode $\varphi_{\mathbf{k}_1}$ is diffracted into the two other modes.

As can be seen from the inset graphs of Fig. 2(a), at low incident beam power the exiting intensity of the mode $\varphi_{\mathbf{k}_1}$ essentially retains its original Gaussian character. However, as the incident power increases, not only does this output intensity diminish in peak value relative to the incident beam, the output also becomes more distorted and the intensity curve tends toward the right side of the crystal. This can be understood by referring back to Fig. 1. Because the mode $\varphi_{\mathbf{k}_3}$ reaches a higher intensity than $\varphi_{\mathbf{k}_2}$, the difference be-

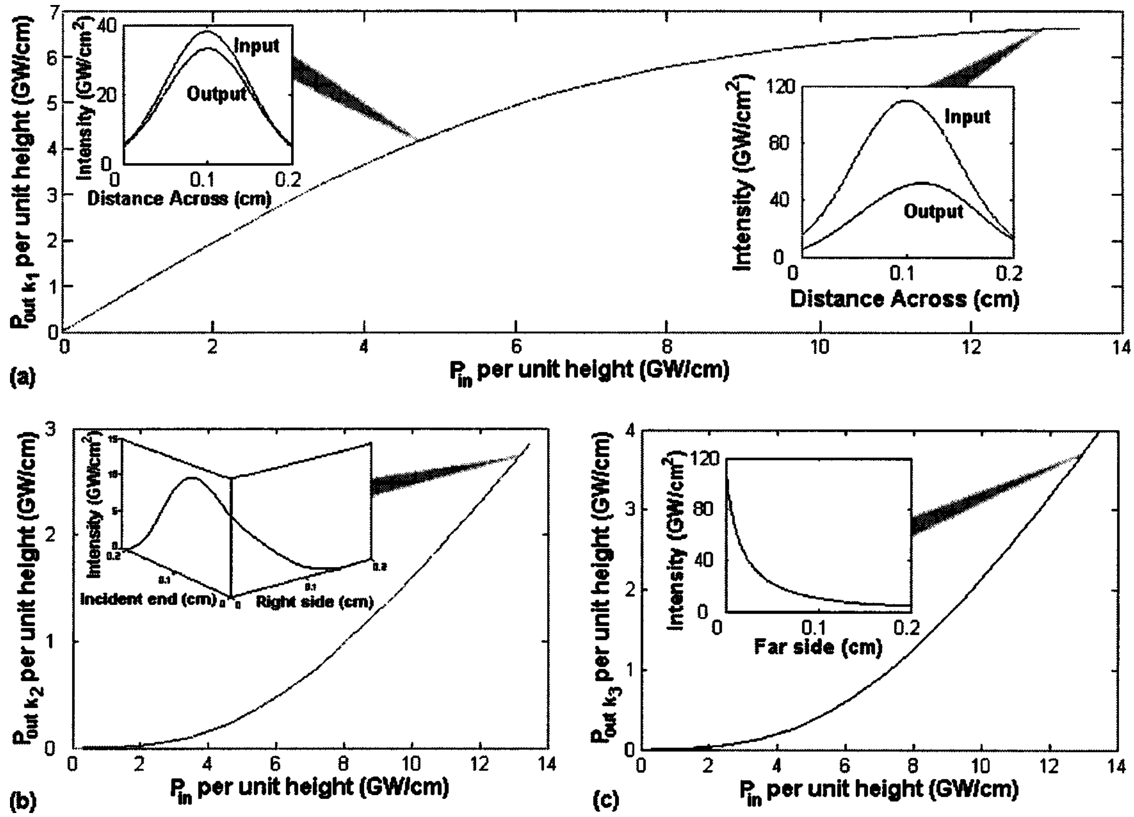

FIG. 2. Output power exiting the crystal with of the modes with wave vectors (a) $\mathbf{k}_1$, (b) $\mathbf{k}_2$, and (c) $\mathbf{k}_3$ versus the power of the incoming beam. The power in (a) approaches a limiting value. The plots inset of (a) display the transverse profile of the input across the incident end and the output across the opposite end. Displayed in the inset of (b) is the profile of the wave with vector $\mathbf{k}_2$ exiting the incident and right sides of the crystal and, in (c), the wave with vector $\mathbf{k}_3$ exiting the left side.

tween the indices of the materials on the left side of the crystal is greater than the difference on the right, so more light from mode $\varphi_{\mathbf{k}_1}$ is diffracted into the other two modes on the crystal's left side than on its right.

V. SUMMARY

In summary, we have developed a method sufficient for obtaining CME's in any Kerr-nonlinear, low-contrast photonic crystal. On applying this formalism to the case of three coupled modes in a 2D lattice, we obtained a set of CME's applicable to any such lattice. Limiting behavior in the specific case of a hexagonal lattice was found by solving those CME's numerically. The full generality of the method awaits to be exploited by, for example, its application to 3D crystals, 1D or 2D crystals with arbitrary field polarizations, or to time-dependent problems.

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[1] N. G. R. Broderick, D. Taverner, and D. J. Richardson, Opt. Express **3**, 447 (1998).

[2] N. D. Sankey, D. F. Prelewitz, and T. G. Brown, Appl. Phys. Lett. **60**, 1427 (1992).

[3] G. Assanto and G. I. Stegeman, Appl. Phys. Lett. **56**, 2285 (1990).

[4] J. He and H. E. Cada, IEEE J. Quantum Electron. **27**, 1182 (1991).

[5] D. Pelinovsky, L. Brzozowski, and E. H. Sargent, Phys. Rev. E **62**, R4536 (2000).

[6] N. G. R. Broderick, D. Taverner, D. J. Richardson, M. Ibsen, and R. I. Laming, Phys. Rev. Lett. **79**, 4566 (1997).

[7] N. G. R. Broderick, D. Taverner, D. J. Richardson, M. Ibsen, and R. I. Laming, Opt. Lett. **722**, 1837 (1997).

[8] D. Pelinovsky, L. Brzozowski, J. Sears, and E. H. Sargent, J. Opt. Soc. Am. B **19**, 43 (2002).

[9] C. J. Herbert, W. S. Capinsky, and M. S. Malcuit, Opt. Lett. **17**, 1037 (1992).

[10] D. Pelinovsky and E. H. Sargent, J. Opt. Soc. Am. B **19**, 1873 (2002).

[11] W. N. Ye, L. Brzozowski, D. Pelinvosky, and E. Sargent, J. Opt. Soc. Am. B **20**, 695 (2003).
[12] L. Brzozowski, V. Sukhovatkin, E. H. Sargent, A. J. SpringThorpe, and M. Extavour, IEEE J. Quantum Electron. **39**, 924 (2003).
[13] J. He and M. Cada, IEEE J. Quantum Electron. **27**, 1182 (1991).
[14] V. Lousse and J. P. Vigneron, Phys. Rev. B **69**, 155106 (2004).
[15] H. M. Gibbs, *Optical Bistability: Controlling Light with Light* (Academic Press, New York, 1985).
[16] H. Lee and G. P. Agrawal, IEEE J. Quantum Electron. **39**, 508 (2003).
[17] N. Akozbek and S. John, Phys. Rev. E **58**, 3876 (1998).
[18] N. Akozbek and S. John, Phys. Rev. Lett. **71**, 1168 (1993).
[19] N. Akozbek and S. John, Phys. Rev. E **57**, 2287 (1998).
[20] C. M. de Sterke and J. Sipe, Phys. Rev. A **42**, 2858 (1990).
[21] S. Pereira and J. Sipe, Phys. Rev. E **66**, 026606 (2002).
[22] C. de Sterke and J. Sipe, Phys. Rev. A **38**, 5149 (1988).
[23] C. de Sterke, D. Salinas, and J. Sipe, Phys. Rev. E **54**, 1969 (1996).
[24] J. W. Haus, B. Y. Soon, M. Scalora, C. Sibilia, and I. V. Melnikov, J. Opt. Soc. Am. B **19**, 2282 (2002).
[25] C. D. Angelis, G. Nalesso, D. Modotto, M. Midrio, A. Locatelli, and J. S. Aitchison, J. Opt. Soc. Am. B **20**, 1853 (2003).
[26] A. Babin and A. Figotin, Waves Random Media **11**, R31 (2001).
[27] J. Poon, E. Istrate, M. Allard, and E. H. Sargent, IEEE J. Quantum Electron. **39**, 778 (2003).
[28] A. Arraf and C. M. de Sterke, Phys. Rev. E **58**, 7951 (1998).
[29] C. de Sterke and J. Sipe, Phys. Rev. A **39**, 5163 (1989).
[30] N. Bhat and J. E. Sipe, Phys. Rev. E **64**, 056604 (2001).